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'rational' minds trying to find their own limits: probabilistic & rationally tuned computer music

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Abstract

This project activates a process of multiple translations and transitions between mathematical concept(s), musical concept(s) and percept(s), highlighting some commonalities between mathematical and musical creativities. This research is mainly focused on the field of computer generated music, specifically where harmony and probabilities play a central role. The thesis presents, in a chronological order, the experiments and compositions prompted by this research, and woven between this, I present extensive analysis and commentary about the influence of works by other composers.

Cover page taken from Catherine Crister Hennix, "Fragments From a Writing of the Unconscious"
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and the constellation of supportive, beloved ones.

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1. Introduction

What interested me most when starting this research was finding commonalities between the mathematical activity and the act of music-making, and to witness how a composer can gradually build a musical edifice from mathematical concepts. This interest was prompted by Fernando Zalamea's philosophy of mathematics, which states:

> What is proper and specific to mathematical activity is [...] to *transit* between a world of ideas of possible, free relations, and a world much richer in determinations, full of precisions and delimitations.[1]

In response to this, I believe the same "transition" is observable in general in the compositional activity and is particularly striking in certain pieces of music, including my own work. This thesis only gives illustrations of this idea by focusing on computer-based music and computer-based analysis of music. The pieces discussed here may appear diverse, and even disparate in their aesthetic interests and philosophical and conceptual background. However, they all share some specifics. Primarily, the pieces emphasize the musical parameter of pitch, a care for harmony and/or rational intonation — while also stressing how these aspects of music can be understood as perceptual phenomena. Secondly, the pieces introduced in this thesis have a strong relationship with computability and/or computational processes. Either these pieces have been generated thanks to computational processes, or analyzed thanks to computational processes. Finally, these pieces all present a degree of randomness, or some of their aspects are left undefined by their composer. Simply stated, a primary interest of mine is to determine whether it is possible to conceive of a computer-generated piece of music devoid of a finite procedure, with no precise set of instructions, which is also still able to articulate a relationship between how (in)determinism can be embedded in (in)computability.

In my own work, these ideas play out in the way I develop music on the basis of computational techniques on the one hand, for instance by writing computer programs for generating conjointly scores for instruments and real-time electronics; while developing a flexibility when notating my pieces and a certain "rigorous," "systematic" openness for their performance on the other hand. I am also interested in the parallelism between the rules of listening and those of composing, with a particular focus on pitch and harmony. Prior to this project, I was already inspired by and playing with certain categories in psychoacoustics: pitch perception thresholds,

[1] Fernando Zalamea, preface to Albert Lautman, *Mathematics, Ideas and the Physical Real*, Continuum International Publishing Group, 2011, p.xxviii.

perception of consonance/dissonance, critical bandwidth. With time, I became more interested in creating in my music an intermediary "situation" for listeners, between the bare perception of pitch and harmonic relationships, and of the music's structure. This interest led me to spend time developing specific musical strategies as well as listening and analyzing my own pieces of music — a process which will become apparent throughout this thesis.

2. Analysis and reconstruction of the algorithm of James Tenney's *Arbor Vitae* (2006)

2.1. A starting point for this research

2.1.1. A significant influence in my musical path

James Tenney's *Arbor Vitae* has been one of the most impactful pieces in my musical development. For me, it illustrates a quintessential juncture between mathematical concepts, musical concepts and percepts. When I listened to the piece for the first time in 2018, I was struck by its clarity and consistency, and its unpredictability. Shortly after, I found out about the general algorithm[2] from which the piece was entirely governed. Developing an understanding of the different perceptual, musical, and mathematical implications derived from the algorithmic procedures used in the piece provided me with a means for gradually incorporating them into my own musical vocabulary.

Specific areas of the piece particularly draw my attention. First, I found out about the possibility of composing "numerically driven music" through *Arbor Vitae* or how to explore the abstractive capacities of computation in relation to sound. This encouraged me to start developing "computational techniques" of composition, i.e. compositional strategies to derive music, sound and scores exclusively from computer programs — something I had never done previously. More specifically, I started delving into probabilistic processes for composing music. Back in December 2018, I already intended to re-construct the algorithm of *Arbor Vitae* for the purpose of producing a new electronic version of it. It is no coincidence that this project was only realized during my Master's project, thanks to the technical assistance of Andrea Vogrig: 3 or 4 years is the time I needed to get some basic ideas beneath *Arbor Vitae*'s algorithm.

Besides the algorithmic aspect of the piece, another essential part of learning from Tenney's piece had to do harmony. *Arbor Vitae* and its complex mathematical edifice encapsulate the composer's exploration of harmony as a perceptual phenomenon, i.e. a framing of musical intervals as periodic patterns, more or less easily grasped by listeners.

[2] See Winter, Michael. "On James Tenney's Arbor Vitae for String Quartet". In *Contemporary Music Review,* Vol. 27, No. 1, Routledge, February 2008, pp.131 - 150.

Finally, the piece inspired me to incorporate physical modelling of strings as a tool for composing. In a metaphorical sense and similarly to the "numeric/mathematical" intuitions one gets when contemplating a tree and getting absorbed by the intricate relation between its branches and roots, I see *Arbor Vitae* as poetically representing an unfolding of the temporal / harmonic structure of a string. In this regard, the poetic presence of a physical model of a string in *Arbor Vitae* definitely (and unconsciously) played a fundamental role in my pieces *for edgars* and *for blandine,* as both were composed for string instruments.

The first months spent on *Arbor Vitae* in autumn 2020 and spring 2021 were based on my listening experience of the piece performed,[3] and my own programming of the piece's algorithm.

2.1.2. James Tenney: musical acoustics & "indeterminacy"

James Tenney was an American composer and music theorist, whose numerous fields of investigation in music and writing include musical acoustics, musical form, perception, graphic notation, computer and algorithmic music, and intonation. Guided by the question of how sound and music are heard, he got involved in computer and algorithmic music from the early 60s. At that time, Tenney was already becoming interested in the cognitive effects of harmonic relationships, developing methods to deal with the gradations of *dissonance* to *consonance*, noise to pitch, and eventually, a more general theory of musical perception. His compositional techniques were continuously informed by musical acoustics, algorithmic procedures and particularly stochastic processes as a means for the composer to expand "variety" in his music[4]. In fact, Tenney's compositions usually illustrate a reconciliation and multiple nuances between random and deterministic processes at different structural levels. This echoes to the second major feature of Tenney's work: his fascination with the notions of *determinacy* and *indeterminacy.* Inspired by the writings and pieces of John Cage and Iannis Xenakis, these two notions and musical influences/ philosophies are often (and equally) present in Tenney's work. Tenney considered determinacy and indeterminacy as very relative: "whatever those polar concepts are, they really are not at opposite ends of some spectrum, but rather points on a circle"[5] that may eventually converge.

[3] Quatuor Bozzini, *Arbor Vitæ,* CQB 0806_NUM, 2008, Bandcamp Audio. https://collectionqb.bandcamp.com/album/arbor-vit . See Appendix 1.1.

[4] See James Tenney, Computer Music Experiences, 1961–1964, Electronic Music Reports 1 (1969): "If I had to name a single attribute of music that has been more essential to my aesthetic than any other, it would be variety."

[5] Charles Amirkhanian, *Morning Concert: Composer Jim Tenney,* interview, KPFMA-FM, 1976, https://archive.org/details/MC_1976_01_12, 63'00"-65'00."

Tenney's very last piece *Arbor Vitae* ("Tree of Life") synthesizes the composer's musical interests and such paradoxical aspects of *indeterminacy*. The piece also mixes opposite polarities and or structures of seemingly opposing logics. Predominantly, *Arbor Vitae* consists in a structural counterpoint between determinism/predictability and randomness/unknowability. From a technical point of view, the piece's algorithm is stunning in its simultaneous high degrees of simplicity and complexity.

2.1.3. Brief overview of the two main components of Arbor Vitae

Arbor Vitae originates from two main components. The first is a simple deterministic algorithm for pitch based on set theory and the harmonic series. The algorithm allows to compute a "tree" consisting of finite branching structures made of a limited set of integer multiples. In the piece, this tree is used as a pitch diagram: a just intonation pitch space, derived from harmonics of B-flat (± 58.27 Hertz), up to its 1331st harmonic. The second component of the piece is a complex non-deterministic algorithm based on probability functions for generating sequences of notes that are both random and harmonically woven. This algorithm organizes the overall temporal and melodic progression of the piece, allowing for a random progression through the harmonic tree consisting of non-repeating sequences of notes. The algorithm designed by Tenney enables various kinds of computations and processes: sequential (re)calculations of probabilities, statistical feedback, multiple tendency-masks (for pitch transposition, dynamics, durations) — resulting in a "complex, time-variant probabilistic scheme."

In 2020-2021, I focused on *Arbor Vitae's* harmonic tree structure as well as a purely technical approach to the piece's algorithm. This focus allowed me to start reflecting on my experience listening to the piece.

2.2. *Arbor Vitae*'s harmonic structure / Tree diagram

Arbor Vitae's tree has its origin in a musical concept of tuning/harmony understood as perceptual phenomena. In Tenney's music, this musical concept may expressed thanks to mathematical structure and concepts based on set theory.

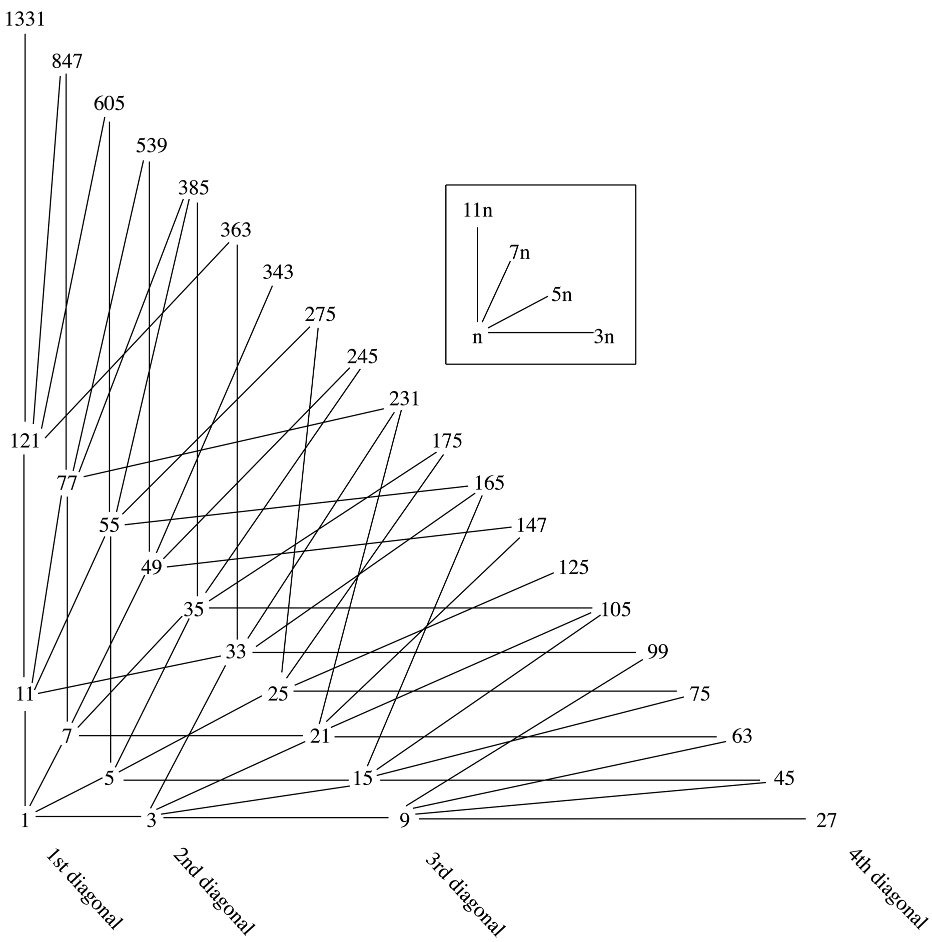

Fig. 2.1: Transcription of *Arbor Vitae's* harmonic structure from Tenney's original notes [6]

[6] See Michael Winter, "On James Tenney's Arbor Vitae for String Quartet". In *Contemporary Music Review,* Vol. 27, No. 1, Routledge, February 2008, pp.131-150.

2.2.1. An extended rational tuning system

First, *Arbor Vitae*'s tree corresponds to an extended just intonation pitch space, forming the harmonic structure of the piece. Whereas intonation is usually defined as "the judicious placement of frequency"[7] or a pitch accuracy, it can also considered as the practice of playing "in tune." There are two common approaches to *intonation*. The first is predetermined, considering every written note as representing a fixed pitch part of a wider, fixed tuning system. The second is more dynamic and dependent on the context of a piece or a performance, for instance where musicians are able to tune themselves to each other according to a tone decided on the spot. *Arbor Vitae* and my work in general relate to the first hypothesis, and specifically the concept of *rational intonation:* musical tunings calculated and related by ratios of integers or whole numbers.

A first specificity of *rational intonation* is that it inherently relates to the harmonic series.[8] As "rationally tuned" intervals produce periodically repeating patterns of sound, rational intonation allows for making a combined periodicity of pitched sounds, and, in brief, to compose harmonic relationships *as* composite periodic sounds. Thus composing with rational intonation entails a care for the specific perceptual effects of harmonic relationships. For instance, a composer may take into consideration how a listener may adjust to degrees of precision in tuning or mistuning in time, i.e. *tuning tolerance*. In Tenney's, this attention to harmony is expressed by representing rationally tuned intervals thanks to what the composer called a *harmonic distances,* laid out in a *metric space* — such as *Arbor Vitae*'s tree. In this sense, *Arbor Vitae* may be partially seen as an experiment with auditory perception and processing of more or less complex tonal/harmonic structures,[9] and/or with the listener's *tuning tolerance* in time. A listener may need more or less time to get "attuned" to the piece's tonal information or *harmonic distances* from B-flat, especially in the midst of the various degrees of timbral consonance and dissonance produced by the instrumentation.

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[7] Clarence Barlow, "On Ramifications of Intonation". In *KunstMUSIK,* No. 16, Cologne, Germany, 2014.

[8] For an extended introduction to Just Intonation, see Marc Sabat and Wolfgang von Schweinitz. "Intonation — An Experimental Application of Extended Rational Tuning." In *Johann Sebastian Bach RICERCAR Musikalisches Opfer 1*, 2001.

[9] Usually, such tree-like diagrams are typically in music cognition research and experiments on auditory perception and human processing of music. See for instance: Jamshed J. Bharucha and Peter M. Todd, *Modeling the Perception of Tonal Structure with Neural Nets,* Computer Music Journal, Vol.13, No.4 (1989) pp.44-53.

2.2.2. A tree echoing to physical models of strings

Being composed as a string quartet, *Arbor Vitae* implies that Tenney paid a particular amount of attention to the phenomenon of string resonance. The effects of rational intonation are especially pronounced on string instruments, producing difference tones and reinforcing common partials. In this regard, I believe *Arbor Vitae*'s harmonic tree and instrumentation work together toward the making of a mass, composite sound of "a string." Subjectively and intuitively, I equated *Arbor Vitae*'s tree structure with a string model drawn from physics. This intuition of mine was due to my previous, biased understanding of "strings," imbued with representations such as the ones bellow, dividing a string into sections at their potential nodes of resonance. Each note of *Arbor Vitae* could be seen a specific node of resonance of a string — if only a finger as thin as a needle that could reach such a level of precision.

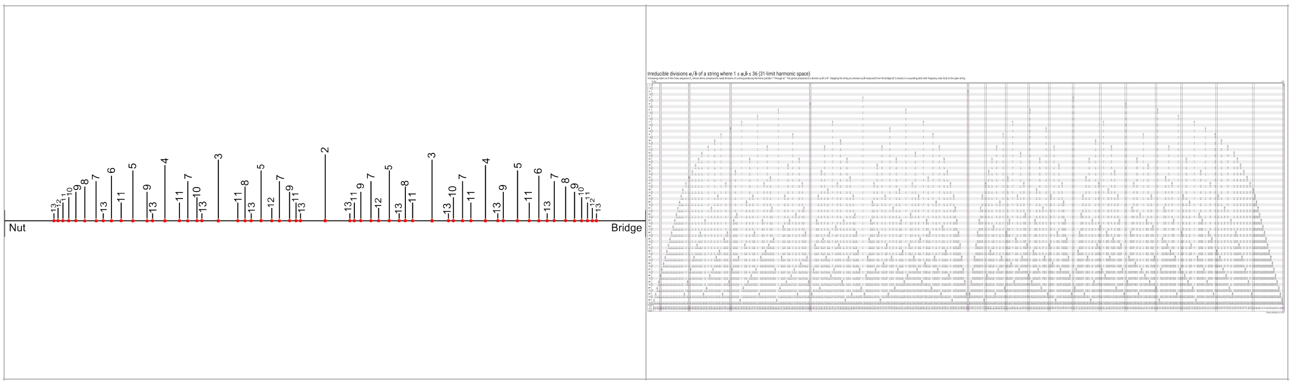

Fig. 2.2: Harmonic Nodes 2 – 13 on a string[10] & irreducible divisions of a string

To me, these string models representations parallel how *rational intonation* systems can appeal to an infinite amount of possible configurations and result in various degrees of precision and extension. Thus many tools and approaches have been developed to circumscribe, map and organize *rational intonation* systems and intervals.[11] In *Arbor Vitae*, this can be done mainly thanks to basic concepts of set theory.

10 The image on the left is taken from Ellen Fallowfield, "Multiphonics: Basics", *Cellomap*, https://cellomap.com/multiphonics-basics/, and the right one from Caspar Johannes Walter, "Meantone Circles, *Texts*, https://www.casparjohanneswalter.de/texts/meantone_circles

11 See for instance the great works of Catherine Lamb, Marc Sabat, Chiyoko Szlavnics, Larry Polansky…

2.2.3. A finite, recursive set of integer multiples

Arbor Vitae's harmonic lattice-structure is a finite set of integer multiples. If these integer multiples seem to be arranged in a dense and intricate network, the way they are initially computed is rather simple.

```
~init = {
   // kernel
   ~k = [3,5,7,11];

   // 1st diag (root)
   ~diag1 = [1];
   // 2nd diag (roots)
   ~diag2 = ~diag1*~k;
   // 3rd diag (roots)
   ~diag3 = ~diag2.collect{|i| i * ~k}.flatten.as(Set).as(Array).sort;
   // 4th diagonal (roots)
   ~diag4 = ~diag3.collect{|i| i * ~k}.flatten.as(Set).as(Array).sort;

   //all diagonals
   ~diags_raw = [ ~diag1 , ~diag2 , ~diag3 , ~diag4];
   ~mult_raw = ~diag1 ++ ~diag2;
```

Fig. 2.3: My translation of Tenney's algorithm into SuperCollider code[12] for computing *Arbor Vitae*'s tree

They are grouped into sets and subsets referred to as *roots* and *branches*. More precisely, *roots* refer to harmonics or integer multiple of B-flat; while *branches* correspond to roots multiplied (once or several times) by 1, 3, 5, 7 or 11. *Roots* and *branches* belong to so-called *diagonals*, i.e. "sets of harmonics (in relation to the fundamental) that have the same number of prime factors (not necessarily distinct)."**[13]** For example, roots having been multiplied 3 times (by 1, 3, 5, or 7) all belong to the 4th diagonal; while the first diagonal consists of the fundamental.[14] This implies a simultaneously arithmetical and harmonic hierarchy between these notes.

[12] See Appendix 1.2.

[13] Michael Winter, "On James Tenney's Arbor Vitae for String Quartet," ibid. p.133.

[14] ibid. p.133: "the 2nd, 3rd and 4th diagonals comprise harmonics with 1, 2 and 3 prime factors, respectively."

2.2.4. A tree implying a hierarchy and interconnectedness of notes

2.2.4.1.Hierarchy

From a musical point of view, *Arbor Vitae*'s harmonic structure implies a hierarchy of notes expressed in the tree diagram (see Fig 2.1) through the presence of what Tenney called *diagonals, roots* and *branches*. First, all the roots and branches are derived from the first diagonal, meaning that *any* note in the piece is conceived in its harmonic relation to B-flat; this defines the main, unvarying and "pervading" *tonal center* in the piece. Hence *any* interval may be represented in relation to the fundamental. For instance, "if roots are prime numbers (harmonics), branches are compound numbers made of the roots (harmonics of harmonics) [of B-flat]."[15] Thus the branch 33 is necessarily a byproduct of the root 11 (3x11), which is a byproduct of 1 (1x11). The further a chosen branch is from the fundamental root on the diagram, the more complex the interval between this frequency value and B-flat. For example, branches on the 4th diagonals correspond to the most complex harmonics of the fundamental root in the piece (605, 847, 1331). Conversely, *roots* and *branches* belonging to the 2nd diagonal correspond to simple intervals or *harmonic distances* that relate to the fundamental such as thirds, or fifths and minor sevenths. Moreover, *diagonals* and *roots* may be seen as "*secondary*" *tonal centers* (this is explained in the subsection 2.3.4.2). For instance, the branch 33 is a byproduct of the root 11, just like the branches 55, 77 and 121. The four branches, harmonically speaking, belong directly to the root 11.

2.2.4.2. Combinatorics

Arbor Vitae's tree diagram evokes combinatorics: it represents all the possible harmonic relationships to B-flat and sequences of harmonic events, i.e. how one element of the tree may lead to another. One of the tree's main features is the strong connectivity of its elements, and this explains why this diagram can quickly entail complex mathematical/harmonic relationships which use very simple computations. For instance, the harmonic 385 connects to 3 harmonics on the 3rd diagonal (77, 55, 35), and these are, respectively, connected to 3 harmonics on the 2nd diagonal (11, 7, 5). Thus, in total, 385 may relate to potentially 6 different "tonal contexts."

Practically speaking, when the elements of the tree are turned into notes, each note may be numerically labeled in according to different harmonic relationships, like shown in the score bellow (Fig. 2.4). A first number may indicate the harmonic in reference to B-flat of the same pitch class as

[15] Michael Winter, "On James Tenney's Arbor Vitae for String Quartet," ibid, p.133

the written note (see upper number next to each note circled in purple on Fig.2.4). This number is the upper one on the score. On the cello stave, these numbers are 63, 27 and 175. Another number may correspond to the harmonic of B-flat that is "greatest common divisor" of successive notes during a portion of the piece (see bottom left of each note circled in blue on Fig. 2.4). On the cello stave, these numbers are 9 for the first two notes and 25 or 35 for the third notes. I call this "greater common divisor" a "temporary tonal center" around which the successive notes of the sequence are derived. Finally, a last number represents the partial in relation to the current "temporary tonal center" (bottom right of each note circled in orange on Fig.2.4). In the cello stave, this number is successively 7, 3, and 7 or 5 for the third notes.

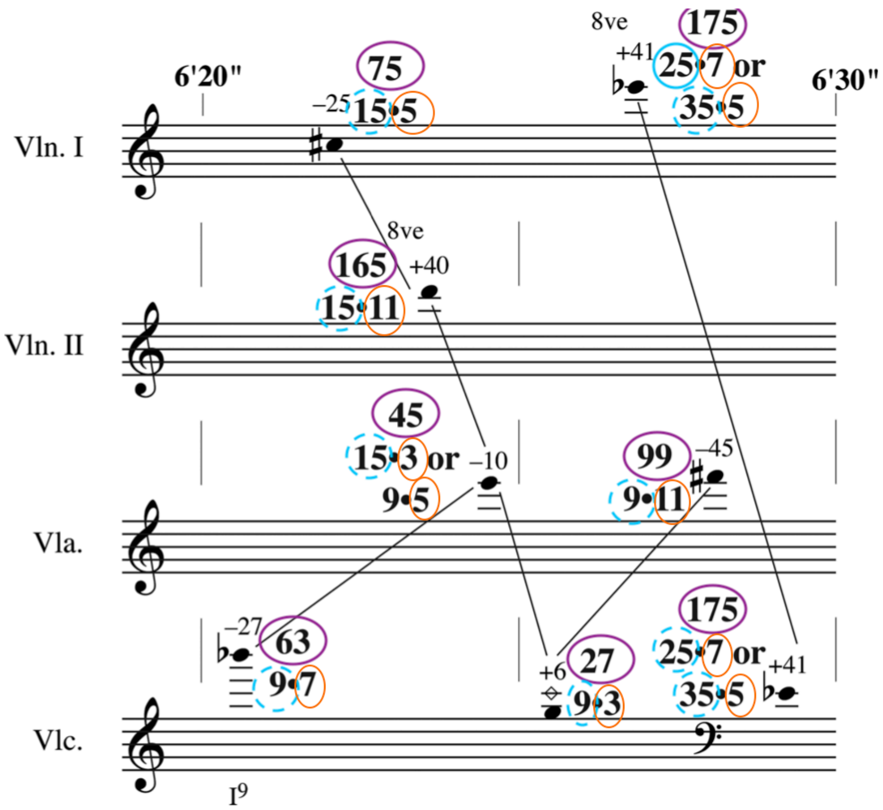

Fig. 2.4: Score excerpt (6'20"-6'30")[16]. The score shows three main roots i.e. "greatest common divisors" or "temporary tonal centers": 9, 15, and 25 *or* 35.

Following this combinatoric feature, "pitches may simultaneously imply more than one tonality since a branch may be a compound integer in relation to the root" and "polytonal harmonies" can emerge since "sets of branches may share a common divisor that is not the [chosen] root."[17] In fact, this combinatoric feature contributes to create a continuity in the piece, illustrated in Fig.2.4 by the lines between the notes that share the same "temporary tonal center." As a listener

[16] Michael Winter, "On James Tenney's Arbor Vitae for String Quartet," ibid., p.137

[17] Michael Winter, "On James Tenney's Arbor Vitae for String Quartet," ibid., p.136

and discoverer of Tenney's music, this may be the element of the piece that I find most fascinating but also understood the least at first. The piece begins with a succession of tonalities, one after the other. Gradually — and rather imperceptibly, it weaves together simultaneous, polytonal harmonies. The piece has no fixed tonality, rather creating a multiplicity of temporary tonalities and "micro-scales" and yet, it grounds the listener in an overall tonal consistency and co-existence of harmonically related pitches. Initially, I assumed this was due to the harmonic tree governing the construction of the work, however, I now realize it is also related to the temporal/melodic organization of the piece.

2.3. *Arbor Vitae*'s non-deterministic algorithm

The second main component of the piece is its non-deterministic/probabilistic algorithm determining the overall melodic and temporal organization of the piece. The algorithm uses the harmonic tree as a sample space for probabilities, attaching evolving probability weights to each element of the tree. On the broad picture, *Arbor Vitae*'s probabilistic scenario is very clear: going through the entirety of the tree diagram in 13 minutes using the most diverse sequence of pitches. However, the algorithm designed for fulling this probabilistic scenario is very complex. *Arbor Vitae*'s computer program consists of hundreds of lines of codes, intermingling sophisticated algorithmic/probabilistic processes and an overarching, temporal sequencing common to these processes. From a strictly programming point of view, the complexity of the algorithm relates to one of the most challenging aspects of probabilistic algorithms: finding the balance between probability weights and events, i.e. the making of the probabilistic bounds needed for an event to happen.

2.3.1. General description of the algorithm

```
strt = 0;
stbr = 0;
initialize rtprobs, rtpsums, and dpsum;
while(strt < 780){
  choose rt;
  recalculate rtprobs, rtpsums, and dpsum
  calculate rdur;
  (re)set multprobs, multpsums, and msetpsum;
  while(stbr < strt + rdur){
    calculate br;
    recalculate multprobs, multpsums, and msetpsum;
    calculate bdur;
    calculate pitch placement;
    assign tone to an instrument;
    stbr = stbr + bdur;
  }
  strt = strt + rdur;
}
```

Fig. 2.5 : Pseudo-code of *Arbor Vitae* written by Michael Winter[18]

In the core algorithm of the piece, *Arbor Vitae*'s harmonic tree is considered as *finite probabilistic space*. Some portions or the totality of the tree are sequentially explored by the algorithm according to a precise and rather simple timeline. Determined, durational bounds (starting and ending points) delineate different "stages" in the pitch selection process, similar to *tendency masks*.[19] These stages consist in randomly selecting pitches from the 4th diagonal and gradually including the entirety of the tree towards the end of the piece.

What interest me most is what happens within each of these "stages" and how Tenney's algorithm generates random sequences of pitches thanks to both random variables and evolving probability weights temporarily attached to each pitch of the harmonic tree. Simplifying *Arbor* Vitae's overall algorithmic procedure to its outmost, the probability of generating a given *root* depends on two main computations. First, at the initialization stage, a random number is generated and its value is compared to each of the root probabilities belonging to one or several diagonals. The chosen root will be the one whose probability value is the closest to this random number. Then, the probability of selecting the next roots depends on the roots that were previously chosen on a

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[18] Michael, Winter "On James Tenney's Arbor Vitae for String Quartet," ibid. p 146.

[19] See Fig.2.6 and the ~rdiag function on my algorithm.

given (or several) diagonals, for a given amount of time. A similar process also takes place for the selection of branches calculated once a root has been chosen. Throughout the piece, one walks down into the tree by recursively evaluating its branches appropriately: each root/branch probability is therefore constantly, sorted, collected, recalculated, and readjusted, as well as the sum of the probabilities of all the branches, roots and diagonals. This aspect of the algorithm is said to be *conditional* on the previous states of the system. In Tenney's work, this procedure of recalculation and comparison of probabilities is often referred to as *dissonant statistical feedback*[20]. *Dissonant statistical feedback* was meant for Tenney to avoid tautologies of pitch classes and, for instance, to ensure that one pitch will not be repeated twice in a row and that a set of pitches are quasi-uniformly distributed throughout a piece.[21] As explained above, *dissonant statistical feedback* is present both for the selection of *roots* and *branches* in the piece.

However, a specificity in *Arbor Vitae* is that, at certain times during the piece, a new random number is generated and this reinitializes all the probabilities of the *roots* and/or *branches* to 0, triggering new *dissonant statistical feedback* processes at different levels of the harmonic tree. This aspect of the algorithm is more similar to *conditional probability* but using a *Markov chain*, i.e. stochastic processes where the probability distribution over future states depends *only* on the present state.[22]

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[20] This procedure was named in this way by Tenney's successors (and I am not sure "dissonant" was the most appropriate terminology as Tenney's algorithm can be abstracted from timbral considerations). See Polansky, Larry, Alex Barnett and Michael Winter. "A Few More Words About James Tenney: Dissonant Counterpoint and Statistical Feedback." In *Journal of Mathematics and Music,* Volume 5:2 (2011): 63-82. This procedure was used by Tenney in previous compositions, like *Changes*. See James Tenney, *About 'Changes': Sixty-four studies for six harps,* Perspectives of New Music 25 (1987), no. 1–2, 64–87, (p. 82): "Just after a pitch is chosen for an element, [the probability of] that pitch is reduced to a very small value, and then increased step by step, with the generation of each succeeding element (at any other pitch), until it is again equal to 1.0. The result of this procedure is that the immediate recurrence of a given pitch is made highly unlikely (although not impossible)."

[21] See Winter, Michael. "On James Tenney's Arbor Vitae for String Quartet", ibid. p.139.

[22] See Wikipedia, "Markov Chain": "it is a process for which predictions can be made regarding future outcomes based solely on its present state and such predictions are just as good as the ones that could be made knowing the process's full history. In other words, conditional on the present state of the system, its future and past states are independent." https://en.wikipedia.org/wiki/Markov_chain

```
/////////////////////////////////////////////////////////////////////////////////////
// calculate (choose_root)
/////////////////////////////////////////////////////////////////////////////////////


~choose_root = {

   ~rdiag = {|time|
      time = case
      { time <  100 }{ 3 }
      { time <  260 }{ 2 }
      { time >= 260 }{ 1 }
      { "error".postln; };
   }.(~strt);

   // rand € R and 0 <= rand <= dpsum
   //~rand = 0.84276;
   ~rand =  rrand(0,~diagonals[~rdiag-1].dpsum);

   // The chosen root (crt) is the rt on the current diagonal (rdiag) with the next greatest rtpsum in relation to rand.
   ~crt = ~roots[~rdiag-1]
   .sort({|a,b| a.rtpsum < b.rtpsum})
   .select({|r| r.rtpsum > ~rand})
   .first;

   // if we are in the first diagonal there is only one root possible
   if(~crt == nil && ~rdiag == 1){
      ~crt = ~roots[~rdiag-1].first;
   };

   // After every crt is determined, the rtprobs of all roots on the current rdiag (except for the root equal to crt) are recalculated
   ~roots[~rdiag-1]
   .select({|r| r != ~crt})
   .do({|r| ~recalculate_root_prob.(r) });

   // Then, rtprob of the root equal to crt is set to 0
   ~crt.rtprob = 0;

   // all the rtpsums and the dpsum of that diagonal need to be recalculated:
   // rtpsums
   ~roots[~rdiag-1].do({|r|
      r.rtpsum = ~roots[~rdiag-1]
      .sort({|a,b| a.rt < b.rt})[0..~diags_raw[~rdiag-1].indexOf(r.rt)]
      .sum({|i| i.rtprob; });
   });

   // dpsum
   ~diagonals[~rdiag-1].dpsum = ~roots[~rdiag-1].sum({|r| r.rtprob});
```

Fig. 2.6: Excerpt of the algorithm for computing a root *Arbor Vitae*'s on SuperCollider[23]

I will not get into the details of how the final notes and durations were derived for the piece, but solely mention this was done thanks to additional semi-determined, semi-probabilistic schemes for pitch-range and durations. The accumulation of these probabilistic processes shows that *complexity* and *random variation* were essential features of the piece.

[23] See Appendix 1.2.

2.3.2. Presence of "efficient" computational techniques

To me, *Arbor Vitae* illustrates how a composer may develop musical processes on the basis of efficient computational techniques and an economy of means — two aspects which are particularly inspiring to me. First, as mentioned above and very classically, the algorithm allows *random variation*. Hence a multiplicity of versions of the pieces may be generated, without changing the structure of the piece. It is also possible to change any of the algorithm's variables, entailing a *scalability* of *Arbor Vitae*'s musical processes, still respecting the piece's core structure. But the most striking features of *Arbor Vitae*'s algorithm are its *recursivity* and *nestedness*, making it particularly elegant, concise and efficient while allowing a complexity in its outputs. The algorithm is based on recursive structures in the form of chains and nested recursion. The presence of recursion in *Arbor Vitae* implies a form of interdependence and *nestedness* between its different routines. First, these recursive structures stem from interdependent modules combined together, such as the generation of random variables, the recalculation of probabilities, the recalculation of the sum of these probabilities. The same interdependent modules are then re-used at different levels, or in different routines of the algorithm. These routines are finally nested inside each other, resulting in the formation of sub-subsequences within a subsequence within a sequence (and so on). For instance, the routine for the selection of branches is nested in the routine for the selection of roots, which is itself nested in the routine for the selection of diagonals.

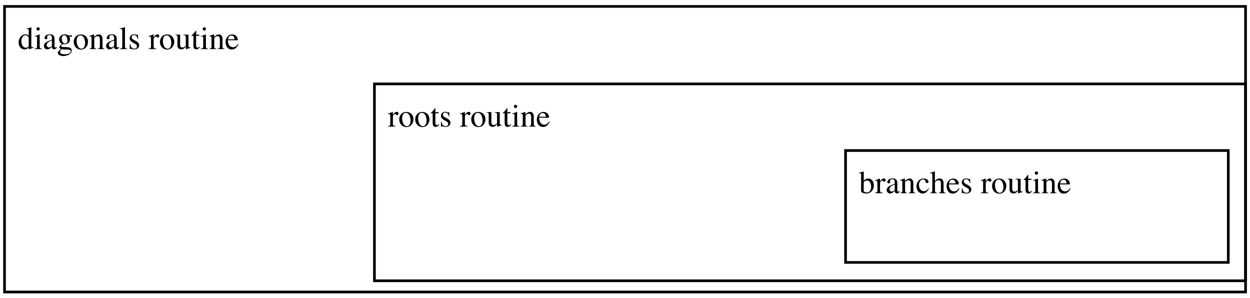

Fig 2.7: a representation of *Arbor Vitae*'s nested sequencing of events

2.4. Some reflections on *Arbor Vitae*'s musical information

Swells and tensions between the different levels of musical information

The overarching sequencing of tendency masks in the piece's algorithm results in the design of a macroscopic *swell*. In Tenney's music, a swell is usually articulated as "an expanding then contracting pitch range, a crescendo/plateau/decrescendo dynamic swell and an increasing then decreasing temporal density"[24]. This "single gestalt" is present on the macro-level of *Arbor Vitae* and applied simultaneously for the piece's different parameters — creating a dramatic tension in the piece, contrasting with the subtleties of its inner developments in terms of intonation. The piece starts from the highest branches of the trees softly played as artificial harmonics. It then gradually includes the entirety of the branches, expanding the piece's pitch range, along with a decrease of durations and a continual crescendo. From the second half of the piece, the pitch range starts to contract (its upper limit going down), while probabilistically travelling more towards the fundamental frequency. Meanwhile, the piece gets louder and its durations shorter. During the last fourth of *Arbor Vitae*, the different parameters return more or less to their initial states at the beginning of the piece: narrow pitch range, high notes, soft dynamics, relatively long durations.

From a listener perspective and on this macroscopic level, *Arbor Vitae*'s sonic information can roughly be perceived as very much predictable and reducible to this simple, distinctive gesture of a *swell*. On the microscopic level though, and as a listener, I have continuously felt challenged in the grasping of the piece's melodic-harmonic information — which very much resonates with the complexity of the microscopic programming level of the piece. To me, all the precision of the piece's nested, recursive programming is conveyed in the complexity of the sonic information of the piece on a microscopic level: the multiple tonalities in the piece are a result of the way each looping sequence of *roots* are constituted from one *branch* after the other, and the way each note follows the other with no repetition, etc. As much as I started understanding the piece's elaborate algorithm, my listening experiences remained limited. *Arbor Vitae*'s micro-melodic/pitch sequences were so complex and intermingled with the timbral richness of the string instruments that they tended to recede into the background of my listening experience. Instead, I was only able to hear more drastic changes of tonalities occurring at turning points in the piece, as well as the overall swell of the piece. This made me wonder if I would get to a different understanding of the piece by hearing a synthesized version of it with sine tones. This curiosity led me to make a sonification of the algorithm that I realized in September 2022.

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[24] Winter, Michael. "On James Tenney's Arbor Vitae for String Quartet", ibid, p.131.

3. *for edgars* (2021)

The first musical outcome of this Master's project is my piece *for edgars,*[25] written for electric guitar and electronics. The composition originated from my basic understanding of some mathematical concepts encountered in *Arbor Vitae* and illustrates ways of applying them to my own work. The piece *for edgars* is one of the first works where I fully integrated rational tuning systems as a part of my compositional interests. More specifically, it is inspired from the formalized structures found in set theory for considering rational intonation. The composition also looks into the use different probabilistic sample spaces for generating pitch material and how their combination may result into mixing different musical notions of pitch intervals.

Many paradoxes confronted me while "imagining" and composing *for edgars* — and most of them express my specific way of engaging with harmony and intervals of pitch. These paradoxes were rendered in contrasting and ambiguous listening possibilities that were inherently proposed by this piece.

3.1. Starting point: rational tunings and septimal minor quarter tone

Rather than following the western tradition of tonal harmony, I have mainly learned about harmony from the perspective proposed by Tenney and other researchers, and based on simple mathematical principles. Such a perspective allows tuning to be central in the composition of a piece. For instance, I am inspired by the standpoint of the American composer Larry Polansky presents a compositional focus that seeks "to allow tuning possibilities to generate and become the form and structure of the work"[26]. I tried to adopt a similar standpoint when composing my piece *for edgars*, which structure originates from the combination of two rational tuning systems.

To ground my first encounter with rational intonation, I developed some mathematical and graphical representations of the two tuning system. These methods helped clarifying the harmonic patterns and sonorities I would eventually decide to use. Doing some back and forth between the graphs and simulations of tuning systems with sine tones were decisive steps for preparing the piece.[27]

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[25] See Appendix 2.1.

[26] Larry Polansky, "A Few Words About Tuning," in *The Just Intonation Issue,* Ed. Nate Wooley, Sound American https://www.soundamerican.org/issues/just-intonation/few-words-about-tuning

[27] See Appendix 2.2 and 2.3.

In *for edgars,* a finite set of frequency values was computed from one "basic" frequency (*f0*) and each value was then derived from a recursive chain of multiplication. This is shown for instance in the following tuning system used in the piece, which I called "tuning (a)" :

- f0 (1/1)
- *f1= f0** (3/2) — *Perfect fifth* in relation to *f0*
- *f2= f1**(4/3) — *Perfect fourth* in relation to *f1 (<=> an octave from f0 (2/1))*
- *f3= f2**(5/4) — *Major third* in relation to *f2 (and from f0 (5/2))*
- *f4= f3**(6/5) — *Minor third* in relation to *f3 (<=> a perfect fifth from f0 (6/2))*
- *f5= f4**(7/6) — *Septimal minor third* in relation to *f4*

Fig. 3.1: Tuning (a) in *for edgars*

Moving on from this, each frequency value was used to tune each open guitar string. Respective subsets of frequencies were then computed from these six initial frequency values, eventually corresponding to the harmonics of each of the open strings (up to their 7th partials).

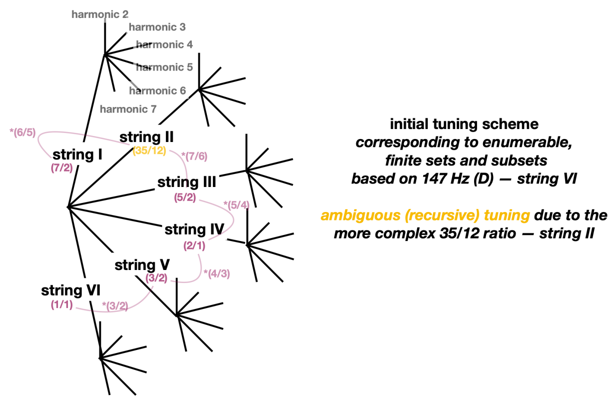

Fig. 3.2 : *for edgars* initial tuning scheme: a computable, recursive, enumerable finite set of pitches (6 open strings and their respective subsets of harmonics)

The piece *for edgars* used two rational tuning system with intervals that produce repeating patterns of sounds of two sorts. When these patterns of sounds are very slow, they give rise to what the composer Marc Sabat calls *harmonic fusion*[28] i.e. a distinctive periodic resonance. Conversely, when these patterns are faster, they may be perceived as regular *beating frequencies*. In *for edgars,*

[28] Marc Sabat and Wolfgang von Schweinitz. "Intonation — An Experimental Application of Extended Rational Tuning." In *Johann Sebastian Bach RICERCAR Musikalisches Opfer 1*, 2001, p.2.

most intervals were very simple, based on fifths, octaves and thirds (see Fig.3.1) and created *harmonic fusion*. But my particular interest in the piece was to enhance the interval between *f4* (6/5) and *f5* (7/6), corresponding to a septimal minor quarter tone (SMQT or the ratio (35/36)).[29] This interval carries a few specificities. First, it creates a periodic patterns between *harmonic fusion* and a sensation of beating. Indeed, this interval may fall into the range of a critical bandwidth[30] and the grasping of this interval depends on the ability for listeners to distinguish timbres in a mass of sound. Then, a SMQT tends to introduce an overall ambiguity in a tuning/scale, disrupting the traditional major/minor polarities. This SQMT entailed important harmonic consequences in the other tuning system used in *for edgars,* tuning (b) :

- *f0* (1/1)
- *f1= f0** (3/2) — *Perfect fifth* in relation to *f0*
- *f2= f1**(4/3) — *Perfect fourth* in relation to *f1 (+ an octave (2/1) from f0)*
- *f3= f2**(5/4) — *Major third* in relation to *f2, (+ an octave and a major third (5/4) from f0)*
- *f4= f3**(7/6) — *Septimal minor third* in relation to *f3 (+a minor sixth (35/12) from f0)* + septimal "color" spread to *f4* subset of overtones
- *f5= f4**(6/5) — *Major Third* in relation to *f4 (*and septimal minor seventh (7/2) from *f0)* + septimal "color" spread to *f5* subset of overtones

Fig. 3.3: Tuning (b) in *for edgars*

Although it shares many frequencies in common with tuning (a), tuning (b) resulted in a "septimal colorization" of its pitch set. This tuning also implies a more harmonic structure in relation to (*f0)* and more beating sensations.

[29] Clarence Barlow's *until…* for guitar and electronics was inspiring in that regard: the piece introduces a difference of a septimal minor quarter-tone in the tune of 3 pairs of strings, subtly enhanced by the electronics.

[30] This obviously depends on which frequencies this interval is being played. If two frequencies separated from a SMQT would be on a very high frequency range, it is more difficult to distinguish them as separate tones.

3.2. Furthering my understanding of sets and probabilistic spaces

Before working on the overall structure of *for edgars,* I wanted to define a simple and clear (and, ideally, effective), general frame of algorithmic, musical structure, a piece "profile" that could potentially be revisited and used for different solo instruments, durations, and tunings. Looking at the sounds, in particular those from the guitar, would only happen at the last compositional stage. Overall, my wish was to compose a gradual *transition* between the two tuning systems presented above. If I was expecting this transition to result in ambiguous perceptual effects for listeners, I also knew there were many ways I could conceive (compose and think of) this *transition*. Following from the mathematical concepts I encountered in *Arbor Vitae,* I wished to delve into the ones which I found most inspiring and accessible, i.e. sets and probability. I thus decided to get back to the very basics of these two foundational branches of mathematics and see how they could prompt new musical ideas — or reveal preexisting, intuitive ones.

3.2.1. Sets (finite, recursive, countable, recursion) / computable functions

While working on the *for edgars*, it was useful to take a fresh look at some basic definitions pertaining to sets and computable functions. This helped me developing efficient approaches for working with rational tuning systems and imagining new possibilities for composing the piece.

A set is a collection of single values. An *enumerable* or *countable* set is one whose members can be enumerated. A set can be *finite* or *infinite:* finite sets have a finite number of elements, whereas infinite sets have an infinite number of elements. *Infinite sets* are the essence of set theory, while *countable* sets are foundational to computability theory. For the purpose of this discussion, I will focus briefly on explaining the latter.

Initially founded by Alan Turing, computability theory allows for the reduction of complex phenomenon to minimal logical constraints. To be more precise, computability theory is concerned with the study of *computable functions* either to enumerate a *countable* set or to approximate *uncountable sets*. Computable functions suppose the existence of a finite, effective procedure, i.e. an algorithm with a precise set of instructions and a finite number of discrete steps, which explicitly state how to compute a function. Given an input of the function domain, the algorithm can return the corresponding output. A fundamental aspect of computability theory lies in the notion of *recursion,* i.e. the process of defining a mathematical object in terms of smaller versions of itself. This is done thanks to *recursive* function, i.e. an algorithm which calls itself during its execution;

hence its outputs depend on previously computed values. Lastly and logically, a *recursive set* entails that there is a computable *recursive* function to list all its elements.[31]

Already in the last chapter, I have shown how *Arbor Vitae* is fundamentally based on computable and recursive functions. The most simple example of recursion was found in the piece's harmonic structure: one single function listed all its elements, simply by calling itself 3 times. Indeed, the set of *branches* were derived from the previously computed set of *roots,* which were also derived from the previously computed set of *branches*.[32] Likewise, *recursion* particularly informs my approach to rational tuning systems and my interest in tuning system that are completely or partially, *recursive sets,* as shown in the tuning systems used in *for edgars* (See Fig.3.2 and 3.3). A much more complex example of recursion was also illustrated with *Arbor Vitae*'s *statistical feedback* probabilistic algorithm that defines the temporal and harmonic unfolding of the piece. When composing *for edgars,* I was not interested in using such complex algorithms based on recursion. However, I knew that the most crucial aspect was finding the right balance of complexity and variety between the harmonic and melodic musical structures throughout the piece. To address this, I returned to the basics of algorithmic randomness and probabilities.

3.2.2. Algorithmic randomness and probability functions

First, I got back to more foundational definitions explaining algorithmic randomness and probabilities and illustrate how the two concepts are intimately connected.

With the use of computers being ubiquitous in society, the word ‘random’ has become a much more common cultural trope. However, *randomness* remains an elusive concept, reflecting different types of conceptual (and mathematical) settings.[33] Randomness is mostly referred to when dealing with *sequences* of datas that have no discernible patterns or regularity. Such random sequences can be *finite* or *infinite*. In this section, I will solely focus on *algorithmic randomness*, that being "individual random *infinite* sequences which can be modelled thanks to the tools of

[31] The elements of the list may be out of order or with repeats. Rebecca Weber, *Computability Theory,* Student Mathematical Library, Volume 62, Providence: American Mathematical Society, 2012, p.97.

[32] My SuperCollider program was a great illustration of this recursion: ~diag4 is computed from ~diag3, ~diag3 from ~diag2 and ~diag2 from ~diag1. See Fig.2.3.

[33] Giuseppe Longo, Catuscia Palamidessi and Thierry Paul "Some Bridging Results and Challenges in Classical, Quantum and Computational Randomness". In Hector Zenil, *Randomness Through Computation: Some Answers, More Questions,* World Scientific, 2011, p.77.

computability theory"[34], i.e. computable functions. To start with, algorithmic random sequences follow *probability functions*. A *probability function* expresses a *probability distribution* or the statistical properties of the random sequence. To each *probability function* is attached a *probability sample space* which corresponds to a set of the function's possible outputs or *events*. In the case of algorithmic randomness, the sample space is a *discrete* one. Additionally, the *probability* of the entire sample space is equal to 1, and for every *event,* referred to as a subset of the sample space, the probability must be comprised between 0 and, at most, 1.

It is my belief that these initial definitions already show how composing music with algorithmic random sequences requires one to "compose a probability," i.e. : to establish the types of probabilistic spaces one wants to work with, and to determine a range of probabilities for each possible outputs of a probability function. Such modelling a musical/algorithmic random process goes against a common, simplistic understanding of "randomness" and "indeterminacy" as "non-intentionality" in the composition of a piece. Similarly, composing with random sequences does not automatically equate with a sequence that will sound superficially "chaotic" or "non-intentional" or "indeterminate" in the perceptual domain — except if this is intended by the composer.[35] In my pieces, starting with *for edgars,* composing with random sequences is intentional: my probability functions are usually very simply and precisely defined, perhaps to the point where one can wonder to which extend probabilities and randomness are relevant tools to generate the material in the piece. Nevertheless, I find the generative aspect and potential of this compositional, and probabilistic process, to be crucial. As a composer, I only specify the big "lines" of a particular scenario, thereby avoiding the need to completely decide every note in the piece. More fundamentally, the generative and random aspect of the work allows me to maintain a level of playfulness, discovery and distance with my material as it is inherently malleable and provisional.

3.2.3. Finite and infinite probabilistic spaces

When I started working on *for edgars*, I was drawn to two types of randomized algorithms, depending on their *probability sample spaces*. The first randomized algorithm I worked into this piece dealt with a *finite probability sample space*. The latter is the most commonly encountered in stochastic music composition and consists of a *finite* and *countable set* of values. An algorithm randomly picks each individual value belonging to a set and then generates a sequence of data following an overall weighing of the selection process, certain values of the set being potentially

[34] Giuseppe Longo, Catuscia Palamidessi and Thierry Paul, *Randomness Through Computation: Some Answers, More Questions,* ibid. p.78.

[35] See Section 4.1 on Iannis Xenakis's *Herma*.

more prone to be picked than others. *Arbor Vitae*'s harmonic tree is typical example of a finite probability sample space whose elements are weighted differently throughout the piece.

The second type of probability, also used in *for edgars,* concerns so-called *infinite probability sample spaces*. The latter have sets of values that delineate the extremes of some metrics. Infinite *sample spaces* are *countable* when they are made of discrete elements that can be counted and there is an infinite number of them (like in the geometric series). What intrigued me most were infinite sample spaces that are *uncountable*. Similar to an infinity of distinct points on a continuous line segment, the elements of such sample space cannot be counted. That said, whether they are countable or uncountable, infinite probabilities are computable: any point or element belonging to infinite sample spaces can potentially be selected by a randomized algorithm. In *for edgars,* I recognized that there was a strong potential to consider rationally tuned musical intervals as possible *infinite uncountable sample spaces,* or "continuous line segments" whose elements could be randomly picked by an algorithm. My interest in this idea also extended from the fact that I had rarely encountered this orientation to musical intervals in other compositions in rational intonation.[36] Additionally, this pointed to a specific relation to musical intervals I was trying to decipher in my own work, which is associated a notion of a frequency continuum and how a listener has a "tendency to treat a range of [pitch] values along a physical continuum as if they were the same until one reaches a point at which the percept abruptly changes."[37] To be exact, I was curious to experiment with *infinite sample space* corresponding to very narrow musical intervals, such as the septimal minor quarter tone, and by so doing, to experiment with listeners's tuning tolerances. In truth, I was also fascinated with the idea of an "infinite" slicing of such narrow intervals because it required a great precision which would hardly be replicable in any acoustic environment and 'sonified' with synthesized sounds.

[36] Except in the work of Chiyoko Szlavnics and in the writings of Catherine Christer Hennix (See Section 5.2)

[37] Siu-Lan Tan, Peter Pfordresher and Rom Harré, *Psychology of Music,* Psychology Press, 2010, p. 98.

3.3. Musical implications of sets and probability spaces in the piece

Revisiting these basic mathematical concepts on probabilities led me to imagine a paradoxical situation in *for edgars*, where finite and infinite sample spaces could be derived from *the same sets of numbers* or frequency values. Yet the cohabitation between two kinds of probabilistic spaces entailed antagonist objectives in the treatment of sets. In a finite sample space, these numbers are considered as potential sole, discrete "outputs" of the probability functions. In an infinite sample space, these numbers constitute the boundaries of the function's uncountable outputs. Inevitably, this made me curious to experiment with the musical implications of this idea in *for edgars*.

3.3.1. Probability spaces in for edgars

3.3.1.1. Finite and infinite probabilities for the parameter of pitch

3.3.1.1.1. A *gradual transition* of tunings (combined probabilistic spaces)

As previously mentioned, my plan was to compose a gradual and probabilistic transition existing between two tuning systems, one that was a result of the combination of the two types of finite or infinite probabilistic spaces. In the way I conceived it, the transition between these tuning systems would in fact correspond to a transition illustrated in the following figure :

tuning (*b*) → *tuning (b)* + *tuning (a)* + SMQT (a)–(b) → *tuning* (*a*)

= finite probability space (1)	*= finite probability spaces (1 + 2)*	*=finite probability space (2)*
harmonic relevance	+ *infinite probability space (1–2)*	harmonic relevance
	harmonic relevance and/or harmonic irrelevance	

beginning of the piece ··········· *middle part* ··········· *end of the piece* ►

Fig. 3.4: A simple scheme of the piece's core idea

The left of the above figure shows the first sample space used in the piece (called "*finite sample space (1)*") which is made of a finite collection of pitches belonging to *tuning (b)*. This means that at the beginning of the piece, the guitar and electronics were exclusively tuned to this specific tuning. Consequently, an overall *harmonic relevance* was highlighted as the two sound layers reinforced the frequencies (and overtones) belonging to *tuning (b)*. Similarly, the end of the piece culminates into *tuning (a)* exclusively.

However, as shown in Fig. 3.4, *for edgars*'s middle part combines a finite sample space ("*finite sample spaces (1+ 2)*") consisting of tuning (a) *and* tuning (b) with an *infinite sample space* which introduces a form of continuum of frequencies between the two tunings *("infinite sample space (1–2)")*. The introduction of an *infinite probability space* entails to "subvert" the rational tunings (a) and (b) and consider how these can be made more dynamic, flexible but also less precise. Hence in the middle of the piece, a *harmonic irrelevance* was introduced since the guitar and/or the sine tones evolved in "out-of-tune" realms, neither fully belonging to tuning (a) nor tuning (b).

To sum up, the last paragraphs touch on the main paradox of *for edgars* which lays in the blending within one single piece of *harmonic relevance* (i.e. a system of rational intonation that is treated as a finite sample space) and *harmonic irrelevance* (partial "blurring" of this rational intonation system when this system is treated as an infinite sample space).

3.3.1.1.2. Practical realization

First, the two pitch sets of tuning (a) and (b) were used as *finite probabilistic spaces*. The individual values of the two tunings were then randomly selected by an algorithm, according to evolving weighing functions (more in Section 3.3.3). This first approach was mainly employed for the guitar part and its fixed score. Then, to include the idea of *infinite probabilistic space* in the narrow range of a semi-minor quarter tone (SMQT), I designed my algorithm so that both the respective subsets of overtones and the frequency values of *f0* (1/1)*, f1 (f0** (3/2)) and *f3 (f2* (5/4))* would randomly modulate in a range of an SMQT[38]. Each of these frequencies *f* would oscillate between an upper boundary equal to *f*, and a lower boundary equal to *f* transposed by an SMQT down. In other words, *f* could be multiplied by a random number comprised between (1/1) and (35/36). Weighing functions were used in the part of the algorithm as well.

3.3.1.2. Finite probabilities for the parameter of rhythm / durations

Regarding meter and durations, the guitar and the electronics followed one unique stochastic model, applied independently to each of the two layers. The same *finite* collection of *durations* (1", 3", 5", 7", 11") were used throughout the entirety of the piece, corresponding to the probability finite sample space for durations. A very simple stochastic algorithm selected individual durations among

[38] Retrospectively, this decision to solely stick to modulations within a SMQT may have impoverished *for edgars*'s potential. It also shows some hesitations in my way of using just intonation.

this set, used for deciding when the guitar should play and when the electronics change in frequency and amplitude. In theory, I thought this would result in an overall metrical/rhythmical consistency between the two layers. In reality, I ended investigating the *indirect interactions* or *loose temporal ties* between the guitar part and the electronics. For the electronics, the computer program followed these durations with an exact precision while the performer was considering them as "averages," thus with more flexibility. By doing so, another aleatoric dimension of the piece emerged: variable, microscopic, random temporal offsets between the electronics and the guitar.

3.3.2.Overall probabilistic scenario for the piece

Weighing functions were another essential tool for me to create a continuous, smooth transition or trajectory between tuning (b) and (a) and give specific directions to the piece's different random processes.

The first probabilistic trend relates to evolving sets of pitches available for the random selection of notes on the guitar part. For making the guitar score, I used a computer program that weighted and selected which string and which note (open string or harmonics) would be played. I gave more weight to specific strings and harmonics throughout the piece (the details of this decision are explained in my description of the code, found in section 3.8). At a later stage, and when possible, I combined these notes into pairs or triads so they could be played as chords by the performer.[39]

The second probabilistic direction given to the piece concerns the gradual transposition of a SMQT down throughout the piece, from (1/1) to (35/36). This transposition mostly affected the electronics but also the guitar part, which was retuned throughout the piece. At first there was greater probability for *f0* (1/1), *f1 (f0** (3/2)) and *f3 (f2* (5/4))* to be multiplied by (1/1), then gradually by a random number comprised between (1/1) and (35/36) and finally by (35/36). The output of this algorithm was used to change the frequencies of the sine tones in real-time during the performance of the piece. Noticeably, the frequency (*f0* *(35/36)) acts as a "bridge" in the piece: it belongs to the two sets of pitches (a) and (b); it is also sustained in the electronics and becomes enhanced in the guitar part throughout.

Finally, amplitude changes are applied to the sine tones throughout the piece. Thanks to other weighing functions, the durations between each guitar note or between each frequency or amplitude change to the electronics are expected to be longer at the beginning and end of the piece and shorter at the middle of the piece.

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[39] This process was an embryonic compositional strategy elaborated a lot further in *for blandine & maciej*.

Further (optional) explanation of each stages of the piece are given bellow.

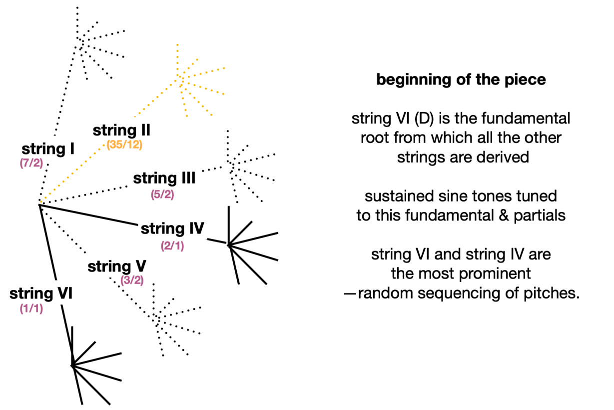

Fig. 3.5 Scheme of the tuning/weighing of pitches at the beginning of *for edgars*

At the beginning of the piece, the guitar's string VI is tuned lower to D. String IV and string VI are the most prominent. More complex melodic and harmonic lines in relation to D occur since there were more chances for higher harmonics to be selected by the computer program. The dotted "strings" on the graph above are thus more of the "background" of the piece. The 16 sine tones are sustained and their frequency values remain constant.

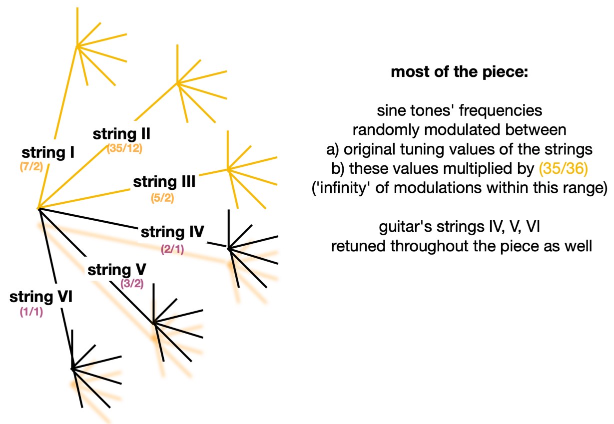

Fig. 3.6 Scheme of the tuning/weighing of pitches as the piece progresses

As the piece progresses, some of the frequencies of the sine tones randomly change between their initial value and this value is then multiplied by (35/36) via a first stochastic routine and a second stochastic routine affects their amplitude. Meanwhile, strings VI, V and IV are retuned one by one a quarter tone down by the performer at three different places in the piece. The guitar part moves towards simpler harmonic relationships in relation with A-47cents. The probability distribution for selecting the strings is a quasi-uniform distribution: all strings of the guitar have a more or less equal chance to be played, just as higher and lower harmonics.[40] For the random selection of durations, shorter durations for the electronics and longer ones on the guitar are preferred when arriving towards the middle of the piece.

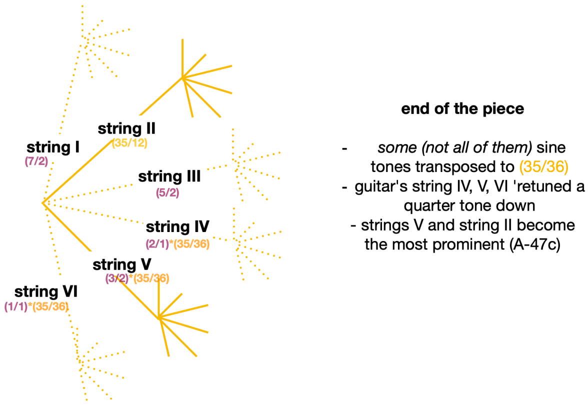

Fig. 3.7 Scheme of the tuning/weighing of pitches at the end of *for edgars*

At the end of the piece, some of the sine tones have been transposed a quarter tone down, string II and string V are the most prominent on the guitar part (both corresponding to an A-47c). Lower harmonics have more chance to be stochastically selected, meaning that the melodic line on the guitar is more harmonically simple.

[40]At first, the harmonically simpler intervals with D are slightly more prone to being picked, and gradually, the harmonically simpler intervals with A-47c are more prone to be picked. But I do believe this idea too timid in *for edgars* but was expanded in *for blandine and maciej* (See Chapter 7).

```
(var openStrings, hmcToFret, stringSelected, harmLimit1, harmLimit2, chosenHmc1, chosenHmc2;

openStrings =[1/1, 3/2, 2/1, 5/2, 35/12, 7/2]; /// D (VI) , A (V), D (IV), F# (III) , A (II) , C (I)

stringSelected= [openStrings[0], openStrings[4]];

  harmLimit1= [7, 6, 5, 4, 1] ;
 harmLimit2= [4, 2, 1, 3];
  chosenHmc1={harmLimit1.wchoose([0.1875, 0.1875, 0.1875, 0.1875, 0.25 ]) };
  chosenHmc2={harmLimit2.wchoose([0.25, 0.25, 0.25, 0.25,  ]) };

 hmcToFret= { |hmc| var fret;
  // fret=0;
  switch( hmc,  |
     1, { fret= 1; "openString".postln},
     2, {  fret= 12;"fret 5 or 12 ".postln},
     3,  { fret= [7, 19]; "fret 7 or 19 ".postln},
     4,  { fret= 5; "fret 5".postln},
     5, {  fret= [16, 9]; "fret 16, slightly before fret 9".postln},
     6,{  fret= 3; "slightly after fret 3".postln},
     7,{  fret= [6, 10, 15]; "slightly before fret 6, slightly before fret 10 ,slightly before fret 15 ".postln},
     8, { fret= [2, 8]; " a quarter after fret 2, slightly after 8," },
     9, {fret=[2, 10]; "slightly after fret 2, slightly after fret 10" },
     // 10, {fret= [2, 6]; "slightly before fret 2, slightly after fret 6" },
     11, {fret= [2, 4.5, 5.5]; "slightly before fret 2, between frets 3 & 4, between frets 5 & 6" }

     );
  ("chosen fret is " ++fret).postln;
};
```

Fig. 3.8: An excerpt of *for edgars's* simple prototype computer program on Supercollider for generating the guitar score

Here are a few explanations about the simple computer program used to generate the score.

- *stringSelected* corresponds to the set of available pitches, to which is attached an array of probability weights (not present on the screenshot).
- Once a pitch has been selected, it can be multiplied by one of the values of *harmLimit1*, corresponding to their harmonic (up until the 7th harmonic). *harmLimit2* allows to transpose this harmonic up till the 4th octave. chosenHmc1 are the probability weights attached to *harmLimit1* and harmLimit2, which are the probability weights attached to *harmLimit2*.
- Throughout the piece, the five variables *stringSelected, harmLimit1*, *harmLimit2*, *chosenHmc1* and *chosenHmc2* evolve.
- A similar operation is applied for the electronics of the piece.

There is no recursion in the program but every function is connected to the other: for example, the function for choosing the octave transposition of an harmonic depends on the function for choosing the harmonic, which depends on the function for choosing an open string.

Finally, the score bellow shows how sticking to numbers only was the easiest notation for both me and the performer.

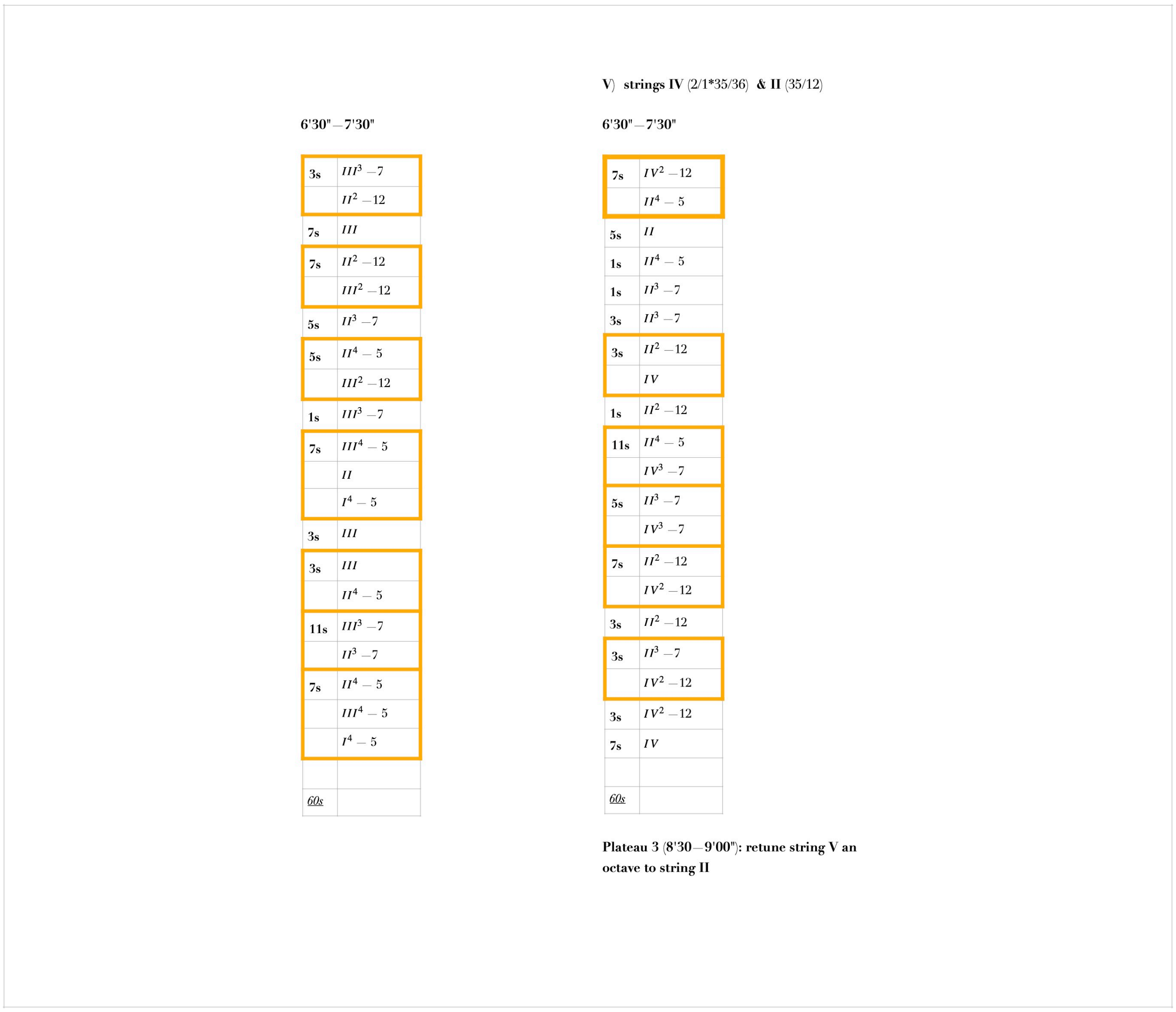

Fig. 3.9: An excerpt of *for edgars's* score[41]

The performer has a certain amount of seconds to play one or several notes. The orange boxes indicate to the performer when to play several pitches together as a cluster or group of notes. They were grouped together as non-repeating clusters of notes — an idea expanded further in my next piece.

[41] See Appendix 2.4.

3.4. Performing the piece

Overall, collaborating with Edgars Rubenis has given me an opportunity to clarify my role in relation to performers, and the agency I expect them to have in my pieces. When asking Edgars about his approach when playing the piece, he answered that his strategy was "ignorance." Never having played in a "classical ensemble" nor "knowing" its rules, Edgars usually "ignores" the other players in improvisation settings. Thus "ignorance" of the rules of classical ensemble playing and ignoring other instrumentalists became Edgars's approach to musicianship in general — and was realized in my piece as well. This unique way of playing meant that Edgars intentionally ignored what was going on with the electronics, avoiding interacting with them as much as possible. In addition, Edgars had never worked with harmonics or just intonation before, and this provided a freshness in the way he related to his instrument.

In my view, Edgars's "ignorance" when performing and his freshness with harmonics contributed to a successful approach for playing the piece. At least, it deeply echoed with the way the piece was composed using random processes, but also the unknown dimension of my compositional process and the recent "leap" I had made with working with just intonation. In brief, writing this piece has shown me how the role knowledge, or lack thereof (ignorance), can have in assisting a performance of this type of music. This led me to begin discovering an approach to my work that touched on certain of limits of "knowledge," specifically when composing, but also while a performer plays — and while a listener listens.

3.5. Ambivalent sounding of the piece & Gestalt principles

From a listener's perspective, *for edgars* and its hesitant sounding raises many questions related to the perception and cognition of auditory/musical information in the piece. On the one hand, the subtleties of certain amplitude changes or frequency modulations are barely perceivable. On the other hand, the piece's musical information is very intricate: frequency modulations of the sine tones occur within an extremely narrow range; frequency modulations between the electronics and the guitar also occur; and different pitch sets unfold and evolve from more simpler harmonic relationships to more complex ones. Reflecting back on *for edgars,* the piece alternates between either very low or very high perceptual and cognitive thresholds for listeners, especially for pitch. The piece met some mixed reactions: some listeners were able to hear most of the piece's features while others were only able to discern a few of them. Even more interestingly, some listeners had an ambiguous response to the piece, being able to grasp and follow the piece's musical information at times, and for some, not at all. These remarks from listeners sometimes made me question whether certain compositional decisions I made were successful. For instance, I wished to understand "why" the piece prompted such drastically different listening experiences among listeners, and, more importantly, to draw some connexions between these reactions and my compositional approach to rational tuning and/or probabilities.

This led me to consider analyzing the piece through the lens of Gestalt theory. This theory seemed applicable as it posits a way to predict thresholds of perception and allows to make better sense of the workings of human cognition. The following Gestalt analysis of *for edgars* helped me gain clarity and announced a different compositional approach for my next piece. In general, three Gestalt principles could be found, albeit ambiguously, in the piece. These were continuity, similarity and proximity.

3.5.1. Abrupt changes and continuous processes (continuation principle)

Originally applied to the visual realm, the continuity principle states that elements arranged as "lines" or "curves" tend to be perceived together as a grouped unit, rather than elements that are not placed on these lines or curves. From this perspective, the sustained, justly tuned tones in *for edgars* could be understood as temporal "lines," evoking the continuity of periodic sounds found in drone music for instance. "Lines" may also be metaphorically found in the piece's linear and gradual processes of transition. Yet, a specificity of my work concerns the simultaneity of continuity in the form of sustained tones and continuous processes (transition), and (abrupt, and often dense) changes (of frequencies and dynamics). For instance, in the middle part of *for edgars*'s*,* the linearity of sine tones tends to be broken apart because of the presence of abrupt changes that are applied to them, or the juxtaposition of the guitar's sharp, short, disruptive sounds that occur on top of them. Nevertheless, as the density of changes diminishes during the piece, these "lines," along with their just intonation signature are found again. One unique regular beating (that corresponds to the interval of a SMQT) is even featured and allowed to grow in amplitude at the end of the piece.

3.5.2. Harmonic fusion, beating patterns, harmonicity (proximity principle)

According to the proximity principle, elements that are close together seem more related than elements spaced farther apart. This principle is particularly helpful when trying to assess the perceptual effects of *harmonic fusion* and beating patterns in *for edgars*. First, the beginning and the end of the piece enhance a sensation of *harmonic fusion* between the tones of the guitar and the electronics, a sensation that goes even beyond a proximity of tones. Secondly, an ambiguous situation occurs in the middle of the piece, combining *harmonic fusion* and *beating patterns*. These beating frequencies unfold within a SMQT, an interval made of very *proximate* tones and already very close to a *critical bandwidth*.[42] Overall, the simultaneity of *harmonic fusion* and *beating patterns* conveys a certain ambiguity or indecisive focus of listening. The latter is either directed to the complete fusion of tones or regularity of the periodic patterns derived from the rationally tuned intervals; or gravitates towards the separation of *proximate* tones, i.e. a variety of beating frequencies contained in a SMQT.

From a compositional standpoint, *for edgars* raises a question about the balance between minute details and variations in pitch, especially with respect to the size of a small interval and the time needed for a listener to grasp these details. In that regard, it is interesting to link the precision of just intonation systems with how our perception and processing of pitch intervals occurs. For instance, we know that it is difficult for any listener to discriminate intervals that do not cross the perceptual boundary between simpler intervallic ratios, as such intervals are too small for our auditory system to distinguish these very detailed variations.[43] This is a reason why most just intonation system set a *prime limit*, i.e. a largest common denominator for all the ratios of integers present in the system. The most common just intonation systems are 2-, 3-, 5-limit; some composers are adventurous to go up to the 19-limit but such complex limits are more rare. Overall, *for edgars* goes up to 7-limit.[44] However, its electronic part may extend far beyond when producing intervals smaller than an SMQT, as these intervals correspond to ratios of very large whole numbers (a random example could be: 1344/1345).

The composer Clarence Barlow worked with these limits extensively and regards that "it is implausible to expect convincing music with an arbitrarily large prime limit without an appropriately developed, well-considered musical grammar for the system that results from it".[45] His comment raises the question about the minimum amount of *harmonicity*[46] needed in a piece of

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[42] A critical bandwidth refers to the narrow range which the ear can perceive two different pitches or intervals as equivalent.

[43] Siu-Lan Tan, Peter Pfordresher and Rom Harré, *Psychology of Music,* Psychology Press, 2010, p. 98: "Frequency is a continuous variable, with an infinite number of possible values, and so it would be inefficient for our auditory system to respond selectively to every possible value."

[44] To be totally accurate, *one* sine tone tuned is tuned to(*f(0)* 11*/2), meaning that the piece has a -11 limit dimension.

[45] Clarence Barlow, *On Musiquantics,* Musikinformatik & Medientechnik, Musikwissenschaftliches Institute der Johannes Gutenberg-Universität Mainz, Report No.51, Mainz, 2012, p.39. Barlow considers that it is mostly relevant to set "the minimum harmonicity to 0.02, allowing to encounter no fewer than 256 intervals in the semitone range of 550 to 650 cents."

[46] Barlow, *On Musiquantics,* ibid. p.61:" harmonicity ('intervallic clarity') stems from the numerical simplicity of the ratio between the two frequencies of an interval, whereby timbre is of hardly any significance; string music, for instance, can be re-instrumentated for winds and/or transposed to extreme registers without losing its harmonic meaning." The harmonicity-algorithm allows to order chords according to their degrees of harmonicity, or 'scaled-harmonicity' and to implement similarities of interval chord structures and proximity in chord progression by adjacent intervals and/or common tones.

music. *Harmonicity*, as defined by Barlow, is a psychological phenomena of "intervallic clarity" for listeners, and more broadly the harmonic sense given to an interval. For Barlow, harmonicity is independent from timbre and distinguished from the physiological and timbre-dependent phenomena of *consonance* or *dissonance*.[47] Rather, *harmonicity* relies on "contextually adjusted listening"[48] and the uniqueness of a listener's attention. *Harmonicity* is also dependent on the context of a given composition, and particularly the time span given in the music for listeners to grasp the composition's harmonic complexity. In the specific context of rationally tuned music, time may allow listeners to develop a certain *tuning tolerance* when confronted with complex harmonic relationships which can then allow them to proceed to a more or less conscious "rationalisation" of the notes and pitch intervals. Interestingly, Barlow considers that in "harmonically more complex music, the minimum harmonicity should be set lower than in more harmonically simple music; lower sensibility results in a higher tuning tolerance." In other words, in harmonically simple music, a very complex tuning may still be understood and clear for listeners, but in harmonically complex music, such tuning would probably not be grasped.

By composing *for edgars* I learned much about this topic, especially when it comes to understanding music through the lens of harmonicity and how harmonicity plays a significant role in the listening experience of my piece. My compositional standpoint was precisely not to fix any minimum "harmonicity" for the electronics in the middle of the piece, and to instead play around with qualities of transparency or a certain blurriness of harmonic relationships. That said, the question remains whether the beginning and end of the piece were long and harmonically simple enough to ground the listeners with intervals that could be easily discerned—namely ones that allow for musical sense to emerge from the microscopic and variable harmonic intervals found at the middle of the piece.

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[47] Barlow, *On Musiquantics*, ibid. p.61. "consonance refers to the ‘smoothness’ (dissonance to the ‘roughness’) of a sound. In the lowest piano octave, a perfect fourth – due to the larger interval size of the critical bandwidth there – can be shown to be more dissonant than a tritone, a whole-tone more dissonant than a semitone. The psychological phenomenon harmonicity originates in the brain’s time-related perception of neuron firing: the physiological phenomenon consonance originates in distances on the ear’s basilar membrane"

[48] Barlow, *On Musiquantics*, ibid. p.61.

3.5.3. Auditory masking (similarity principle)

The similarity principle states that we tend to group element together when they appear to be similar. This principle finds a specific parallel in *for edgars* with its intermittent effect of *auditory masking*. Masking occurs when the perception of one sound is affected by the presence of another sound.[49] Masking acts on our perception of timbre, in this case the combination of sine tones and the guitar, one masking or being absorbed by the other. Partial masking by one sound on another results in a reduced perceived loudness of the weaker partial. It can be "a very complex process, where amplitude and frequency of both masking and masked sounds interact in ways which are far from linear, and are also strongly affected by phenomena such as beats and difference tones."[50]

Initially, considerations about auditory masking influenced the composition of *for edgars*'s spatiality. This is most apparent at the beginning and end of the piece, as I intended to emphasize the timbral separation between the guitar (having its own amplifier on the right side of the room) and the electronics (two loudspeakers, left and center). Furthermore, as the piece unfolds, my plan was to increase the volume of the central loudspeaker playing the sine tones, which have the lowest frequencies, to create a strong auditory masking effect with the guitar sounds. Though the reality was that the perceptual distinction between the two sound sources in the piece was, at times, very strong, and far blurrier at other times. Masking is thus highly unpredictable in *for edgars* and is like this for several reasons: the harmonics of the guitar are sometimes weaker than the sine tones, and, conversely, the sine tones can sometimes disappear in the spectral richness of the guitar's open strings. Moreover, the sharp attacks of the guitar sounds either get absorbed into the sine tones or come suddenly to the foreground. As a result of these circumstances the timbral merging between the two layers appears and disappears in unforeseeable ways.

Last but not least, I realized in *for edgars* how timbral masking influences the perception of the just intonation's periodic signature of the piece, or its beating effects. In theory, the psychological phenomena of *harmonicity* or "intervallic clarity" is independent from timbre. But it is also true that a clean timbre serves (or suggests) a clean intonation. In other words, the cleaner the timbre and the simpler the rational harmonic relationships are the more graspable they will be for listeners. This is illustrated at the beginning and the end of *for edgars,* where the periodic patterns of tunings (a) or (b) come out well and are coherent with the timbral simplicity of sine tones and the guitar harmonics. The two sound layers reinforce an overall harmonic simplicity or each other in

[49] Wikipedia, *Auditory Masking*, https://en.wikipedia.org/wiki/Auditory_masking

[50] Pedro Manuel Branco dos Santos Bento, *The Harpsichord: Its Timbre, Its Tuning Process, and Their Interrelation*s, University of Edinburgh, 2013, p.58.

their "in-tune-ness," largely because the sine tones are sustained in a "clean" intonation thus acting particularly like an anchor for listeners, providing a stable, sonic context to hear the tuning of the piece. However, when the intonation is less clean in the middle of the piece, it is as if the guitar and sine tones start working against one another, "unmasking" each other's presence and their differences of tunings. The sine tones also start misleading the listener's ear as the sonic context they initially provided breaks apart.

These last few remarks reveals another paradox present in the piece. Besides the clashing of what I called earlier *harmonic relevance* and *harmonic irrelevance* in the structure piece, there is a clash between "clean" *timbre* or transparent sound sources like sine tones and the harmonics of the guitar, with "unclean" intonation. I found these unfamiliar, strange musical territories quite interesting and explored them further in the next piece I will discuss, *for blandine and maciej*.

3.5.4. Melodic contour & Uniform probability distribution (ergodicity)

One aspect of the piece which may be the easiest to process for the listener concerns grasping the work's different macro-*sequences*. Specifically these are so-called moments of "plateaux" where the guitar is retuned and the electronics are also kept steady, primarily for the purpose of announcing new stages in the piece, where new sets of pitches and harmonics are introduced on the guitar. Due to the nature of the available pitch sets, and how they evolve throughout the piece, it is hypothetically possible to hear more complex harmonic and melodic relationships on the guitar at the beginning of the piece, and a more simple harmonic cloud in the middle. But within each of these macro-sequences, the piece's melodic contour is difficult to grasp, probably because of the use of random processes for pitches solely using a quasi-uniform probability distribution throughout the entirety of the piece. For each section or each available pitch set, the pitches had a more or less equal chance to be selected by the algorithm. As a result, when considering *for edgars* as a whole, one may mostly retain an overall ergodicity in pitch: everything is "different" yet everything sounds the same. I came up with the conclusion that composing/shaping the probability distribution for weighing the random selection of notes in the piece seemed crucial from a listener's perspective.

Conclusion

In retrospect, there was room for progress in the piece. For instance, I could have gone much further in the variety of harmonic material in the piece, or incorporated 'harmonicity' as a parameter in the piece. I could also have counterbalanced better the degrees of complexity of harmonic relationships with degrees of "timbral clarity" — especially in my way of using the sine waves in the piece. Finally, playing with auditory masking could have been beneficial, for instance by drastically separating the guitar part from the electronics at some moments in the piece. These intuitions were eventually explored in my next piece *for blandine & maciej*. In the meantime, I was curious to see how other composers had dealt with rational tuning systems and probabilities in their music; and composers I found influential in this regard will be examined in the next chapter.

4. Pioneering "indeterminate" pieces examined through computability and computationality

Before discussing more recent music, it is interesting to go back to John Cage's *Number Pieces* (1988) and Xenakis's *Herma* (1965).[51] Both of these works are so-called "indeterminate" pieces, applying aleatoric procedures for generating pitch material. I will present the two pieces in this section for the sake of illustrating the process of translations and transitions between mathematical concepts, musical concepts and percepts — from an analytical perspective. Indeed, what I find most interesting is that these two pieces have both recently been analyzed through the lens of computability and/or algorithmic information theories and Gestalt theory. Thanks to the specific methods used, the analysis of the two pieces shed new light on the paradoxical aspects inherent to both of them. This includes *Herma*'s effects on listening in relation to Xenakis's compositional intentions and methods; and, with respect to the *Number Pieces,* more recent analysis adds nuances to Cage's compositional standpoint and personal approach to understanding harmony. Further, by making Xenakis and Cage's original compositional intentions "revisable," these analysis result in a reversal of a commonly held outlook towards their pieces and how their aleatoric dimension for generating pitch material may be understood.

To sum up, it is my hope to underscore throughout this section the existence of dynamics of opposite logics and polarities, such as determinism and indeterminism, simplicity versus complexity, and composing and analyzing. All of these are important, and in my opinion are present in *Herma,* the *Number Pieces,* their analysis, as well as my own work.

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[51] See Appendix 3.1 and 3.2.

4.1. Iannis Xenakis's *Herma,* Gestalts & Computable analysis

For me, *Herma* has much to do with the workings of a "specific" creativity consisting of an interplay between knowledge and ignorance. Xenakis's adventurous and brave compositional process in *Herma* encapsulates the imperfect emergence of knowledge and composition/music — or how one has to work constantly in the unknown in order to actually learn from and under these conditions. *Herma* is inspiring for me for this reason. It provided me with the courage to develop and to compose in a similar intermediary, dynamic, and creative state, oscillating between knowledge and ignorance. I also consider *Herma*'s experimental spirit as a template; one that forgives any erroneous ideas about one's work.

4.1.1. General presentation of the piece

Written in 1961, the piano piece *Herma* is a turning point in Iannis Xenakis's musical path. Besides being his first piece for solo instrument, the piece departs from the solely stochastic means of composition previously he used. *Herma* instead raises compositional questions having to do with the potential to simply "deduce" music from mathematics and mathematics from music. Xenakis started from concepts drawn from set theory to derive the structure and score of *Herma*, applying logical operations to its musical parameters — mainly pitch. *Herma* illustrates the composer's notion of *Symbolic Music*: a music that would allow a listener to "reason by pinning down our thoughts by means of sound"[52] —in other words, a music that conveys a complete transparency between mathematical concept(s) and percept.[53] As explained in his writings about the piece, Xenakis was aiming at a perfect consistency and correspondence between the three poles of mathematical concepts/model (sets and stochastic processes), musical concept (mainly related to pitch), and how music is heard by listeners. Hence *Herma* is an interesting case study to examine if such a complete transparency is actually valid/tenable.

In reality, the piece revolves on a coexistence of both transparency and ambiguity. On the one hand, *Herma* is undeniably numerically driven: its compositional methods originate from rigorous mathematical concepts and the complexity of the music when heard could be spontaneously interpreted as resulting from the mathematical dimension of the piece. Hence on a

[52] Iannis Xenakis, *Formalized Music, Thought and Mathematics in Music*. Harmonologio Series No.6, Revised edition, Pendragon Press, Stuyvesant NY, 1992, p.172

[53] This reminds of Fred Lehrdal and Ray Jackendoff's music cognition model where the existence of an "ideal listener" is assumed to justify the affinity between the rules of listening and those of composing, through an objective structure which can be successfully communicated through the music. See Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music,* MIT Press, 1982.

superficial level, the piece conveys a transparency between mathematical concept(s) and percept. However, when examined closer, one realizes that the compositional methods used by Xenakis entailed some aesthetic and technical contradictions in relation to his initial intentions, touching upon the way the piece was made and the listening experiences it was supposed to provide. Thus *Herma* illustrates an intricate relation between mathematical concepts, musical concepts and percept: it is transparent and it is not and this, in my opinion, makes it a fascinating work to analyze.

4.1.2. Xenakis'*s compositional methodology and musical strategies in* Herma

4.1.2.1. Compositional methodology: a set theoretic," imaginary experiment in outside time"

First, I wish to highlight the existence of a specific compositional methodology in *Herma* — already encountered in *Arbor Vitae* and that I am also investigating in my work. Although *Herma, Arbor Vitae* and *for edgars* have (very) different musical grounds and motivations, they share a methodology that implies the following steps. The first step takes place prior to the musical time proper. It consists in defining the set of all potential pitches in the piece and then all the subsets of pitches and the (mainly combinatoric) operations that can be applied to them. The following step consists in presenting the results of the set operations dynamically through probabilistic processes, i.e. sequentially organize pitches into random sequences. Finally, these random sequences are eventually transformed into sonic events.

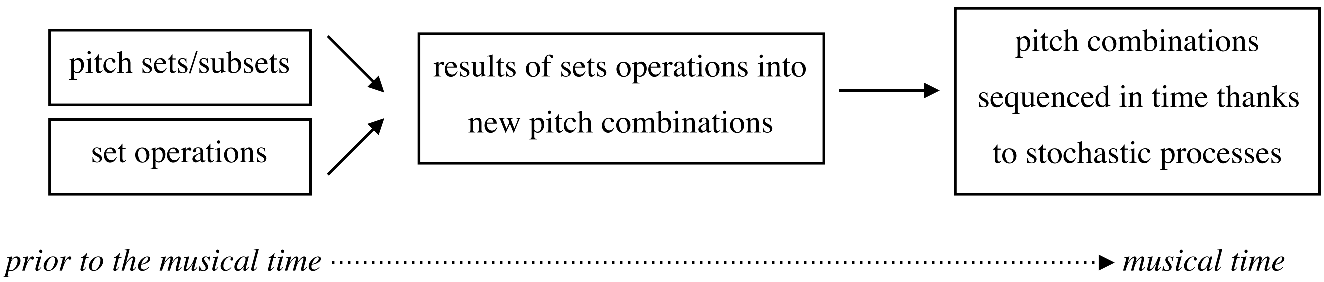

Fig 4.1: A methodology common to Xenakis, Tenney and I

Indeed, in *Formalized Music,* Xenakis explains having conducted an "imaginary experiment" to compose *Herma,* abstracted from any musical temporality, unfolding in what the composer calls the *outside-time*.[54] The experiment was supposed to embody basic concepts of set and computability theories. The composer first defined, stated and sketched a map of "*state classes*

[54] Iannis Xenakis, *Formalized Music, Thought and Mathematics in Music*, ibid. p.166 "an algebration of sonic events that is independent of time (algebra outside-time)" VS "an algebration of sonic events as a function of time (algebra in-time)."

of sonic events"[55], corresponding to the set theoretic features of *Herma*. Typically, Xenakis adopted as a universal pitch set *R,* the set of all 88 keys on the standard piano keyboard. He then derived three subsets of pitches *A, B*, and *C* from *R,* to which he applied specific elementary set operations: intersection, union, complementation. The results of these operations between the three pitch sets define the macro-structure of the piece, each section corresponding to a specific mathematical relationship. Then, the individual pitches in each section proceeds from stochastic processes: the pitches are picked at random from the given sets and/or their combinations, without registral preferences.

Herma's specificity has, on the one hand, much to do with the roles Xenakis gave to musical time ("in-time") and listening, and on the other hand, with the musical strategies he used.

4.1.2.2. Role of mathematics in *Herma*

4.1.2.2.1. Questioning the possibility of an "immediate comprehension" of mathematical concepts through sonic events

When reading Xenakis's original writings about *Herma*, one can discern specific functions attributed to sonic events and perhaps listening by the composer. It seems that listening to sonic events is considered by Xenakis as a direct, immediate way of knowing or having access to the mathematics beneath the piece. At least, this is my take on Xenakis's idea when he writes that *Herma*'s pitch sets and their mathematical operations applied to them were meant to be "deduced mentally by the observer"[56]. As Xenakis invites the listeners to undertake an "intellectual task" of looking for such relations while simultaneously listening to *Herma,* it seems that listening should be equivalent to an "*immediate comprehension*"[57] of a *conceptual,* "*deduced*" knowledge for the

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[55] Iannis Xenakis, *Formalized Music,* ibid., p.171.

[56] ibid.

[57] Iannis Xenakis, *Formalized Music,* ibid pp.171-172: "an observer must undertake *an intellectual task in order to deduce from this both classes and operations. On our plane of immediate comprehension, we replaced graphic signs by sonic events*. We consider these sonic events as symbols of abstract entities furnished with abstract logical relations on which we may effect at least the fundamental operations of the logic of classes. We have not allowed special symbols for the statement of the classes; only the sonic enumeration of the generic elements was allowed (though in certain cases, if the classes are already known and if there is no ambiguity, shortcuts may be taken in the statement to admit a sort of mnemotechnical or even psychophysiological stenosymbolization). We have not allowed special sonic symbols for the three operations which are expressed graphically by " +, - ;only the classes resulting from these operations are expressed, and *the operations are consequently deduced mentally by the observer. In the same way the observer must deduce the relation of equality of the two classes, and the relation of implication based on the concept of inclusion*. The empty class, however, may be symbolized by a duly presented silence." My emphasis.

composer. Evidence of this is vaguely implied by Xenakis when he writes that "we can reason by pinning down our thoughts by means of sound."[58]

However, this is also where I begin to find Xenakis's writings (and overall concept of "Symbolic Logic") puzzling. What I find problematic is not the idea of deducing some knowledge from music *eventually* but the potential confusion between direct knowledge (perception) and deduced, indirect (intellectual) knowledge. In fact, Xenakis seems to suggest the possibility of an immediate *and* deduced knowledge through the listening to sounds all at once when he mentions the equivocal idea of "a sort of mnemotechnical or even psychophysiological stenosymbolization"[59]— which is disputable. Listening is a mode of perception or sensory awareness which may lead to immediate comprehension. In that regard, I find the Buddhist epistemology of the Mind helpful. The latter refers to perception as direct knowledge, and thus acknowledges that the perception of sounds *is* direct knowledge. Yet for it to be immediate, such perception or direct knowledge cannot be at the same time produced by inference, deduction or reasoning, understood in the Buddhist epistemology as indirect knowledge or mental impressions. In other words, perception cannot lead to mentally inferred knowledge. This idea, which seems to go against Xenakis's premises in *Herma,* is summarized in the *Sanskrit Algebras* of Catherine Christer Hennix (another important composer and figure presented in Chapter 5.2), when she writes: "Pure sensations stand out alone in containing no constructive coordination. It is uncognizable by logical methods."[60]

4.1.2.3. Mathematics as tools for developing compositional strategies

Putting aside Xenakis's premises of "Symbolic Logic," I would say that the role of mathematics in *Herma* is the same as in any other numerically driven music: mathematics just acts as compositional "tools," used to prompt some perceptual effects, convey specific mental impressions to listeners, and find new ways of making music. For instance, the random ordering of the individual pitch sequences he used were designed to avoid any melodic or harmonic patterns that could interfere with the listener's perception of the pitch sets and the logical operations applied to them. Besides the ordering of the individual pitches, the "intensities and densities, as well as the silences, are meant to help clarify the levels of the composition."[61] All these musical strategies imply an overall technical difficulty in the piece, resulting into new forms of gesturalities on the

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[58] ibid.

[59] Iannis Xenakis, *Formalized Music,* p.171.

[60] Catherine Christer Hennix, *Poësy Matters and Other Matters,* Vol. II, New York: Blank Forms Editions, 2019, pp.80-82.

[61] Iannis Xenakis, *Formalized Music*, ibid., p.175.

side of the performer. These gestures are technically very challenging and original, like the way a performer must play in all the registers of the piano at an extremely high speed or very precise rhythms, constantly alternating between very big dynamic ranges or timbres (staccato/pedal).

4.1.3. Computable and Gestalt analysis of the piece

When listening to *Herma,* the piece has a perceptual clarity of its own, characteristic to Xenakis's style (and perhaps other serialists of that time). The piece sounds at once hectic, dense, but more importantly: clear. At first glance, *Herma* even conveys a clear stochastic "form"[62] in listening, indicating that some aspects of the relation between some of mathematical concepts beneath the piece and its percept are preserved. Yet, despite its superficial, external characterization of a random process, the piece presents many aspects of mathematical inconsistency and aesthetic contradictions. When giving the work a closer look, one may get curious to know whether Xenakis's musical strategies were:

- mathematically rigorous (translation from a mathematical to musical concepts);
- "musically effective" in the sense of guiding the listeners to grasp the main musical concepts beneath the piece (translation from musical concepts to the music *as heard*). This particularly applies to the grasping of pitch sets and their sequencing in the piece.

The above questions have been thoroughly exposed by the composer and the music theorist Robert Wannamaker, specifically in his meticulous study of *Herma,*[63] which certainly helped to complement some previous reflections I made about the composition *for edgars*.

4.1.3.1. Computable analysis (translation between mathematical and musical concepts)

Wannamaker's computable analysis reveals more broadly the tension between using mathematics as tools for music-making, and/or the consistency with which these tools are used. To me and from a general perspective, such tensions invite a composer to shape their own view on the role of mathematics in their music.

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[62] Expression taken from Sam Goree, *Structure and Randomness in Iannis Xenakis' Analogique A,* Musical Capstone Thesis, 2017, p.34: about *Analogique A/B* where "the micro-level of the piece was [most likely] composed intuitively, without the use of stochastic techniques, to fit a stochastic form."

[63] Robert A. Wannamaker, "Structure and Perception in *Herma* by Iannis Xenakis". In *Music Theory Online,* Society for Music Theory, Vol.7, No. 3, May 2001, [1.3].

With this in mind, in theory and ideally, *Herma*'s musical structure should stand for a mathematical demonstration related to sets, set operations and stochastic processes. All these operations and processes are perfectly computable and easily translatable into musical material. However, and contrary to his writings, Xenakis did not rigorously follow the mathematical model from which the piece was supposedly built upon.

To begin with, the validity of some of *Herma*'s set-theoretic and stochastic features is open to doubt: both the set-algebraic computations and stochastic processes exposed in *Formalized Music* are not always in line with the actual score of *Herma*. Besides, what seems to be an arbitrary choice made by Xenakis concerning the three sets of pitches *A, B* and *C,* reveals inconsistencies that, according to Wannamaker: "appear to persist throughout the score based on the set-algebraic computations…in which significant disagreement between computed sets [of pitches] and their representations in the score is repeatedly observed."[64] By this statement, it appears that certain pitches are, for instance, present in numerous sections when they should be excluded from them if one would strictly follow set-algebraic operations described in *Formalized Music*.

In addition, the "stochastic processes" mentioned by Xenakis to generate melodic sequences from the pitch sets raise other problems, and these issues exemplify the juncture between computability and the craft of composition. As compositional "tools," stochastic processes come into contradiction with Xenakis's musical idea of demonstrating "completed" sets of pitches. Indeed, from a statistical point of view, it would be extremely rare, if not impossible, that uniformly distributed stochastic processes would be able to display the complete set of 88 values in such short sequence of datas (see exposition of the piece: 88 different pitches out of 205 notes).[65] This shows that the resulting sequences of pitches in the piece are in fact highly improbable, suggesting Xenakis partially composed, or wrote, the notes himself. But whether or not Xenakis did write these notes himself, what I find even more interesting is that, *in any case,* certain pitches are never sounded in the piece. To me, this hints that Xenakis did not consistently follow the mathematical model he was after, nor his initial musical idea of covering the entirety of the 88 pitch set.[66] He just did something slightly different.

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[64] Robert A. Wannamaker, "Structure and Perception in *Herma* by Iannis Xenakis", [4.12].

[65] Wannamaker, "Structure and Perception in *Herma* by Iannis Xenakis", [4.1]: "a computer simulation employing ten million trials of the pitch lottery has shown that an average of 446 draws are required to obtain all 88 possible pitches (with a standard deviation of 110 draws), and that the probability of obtaining all 88 pitches in 205 or fewer draws is extremely small (about 3 in 100000). "

[66] ibid, [4.2]: "if the object of the composition is to demonstrate certain set-theoretic facts, then a complete representation of the sets of interest would seem to be a necessary prerequisite".

4.1.3.2. Gestalt analysis (translation between musical concept to percept)

If focusing on Xenakis's musical concept of presenting specific sets of pitches to the listeners, Wannamaker believes that "the macro-structure of the pitch sets [is] not the primary determining factor in low-level temporal gestalt formation."[67]

Certainly, perceptual units at the sectional level are easily demarcated thanks to radical changes in dynamics (silences), temporal densities (varying paces) and timbre (pedal). However, these drastic changes delineating *Herma*'s sections do not necessarily help discriminate the different pitch sets within each section. Similar to my experience in *Arbor Vitae, Herma*'s density of musical information on the microscopic level causes a discrepancy between the set-theoretical structure of such a piece and the structures a listener can perceive and retain in their memory.

Then, and this echoes an aspect of my piece *for edgars,* the application of stochastic processes in the piece but also the distribution of notes among all the registers of the piano result in an *ergodicity* in pitch materials: they sound statistically homogeneous when considering *Herma* as a whole. This means that every sample of a given sequence of *Herma* would be equally representative of the entirety of the piece in regard to pitch. Thus, for Wannamaker and from an aesthetic point of view, this overall perceived ergodicity in pitch materials contradicts Xenakis's initial compositional intentions.

Just as I discussed in *for edgars, Herma* gives an example of challenges encountered when composing with this specific "methodology" I presented at the beginning of Chapter 4: sometimes, the methodology simply does not work according to what a composer had planned.

[67] Wannamaker, "Structure and Perception in *Herma* by Iannis Xenakis", ibid, [3.10].

4.1.4. Conclusion

To me the conceptual and perceptual paradoxes or conflicting aesthetics in the piece are not flaws. On the contrary, they participate in sculpting an overarching heterogeneous compositional idea and musical effect. Xenakis's style is not found in one consistent application of a single theoretical idea, but in a multiplicity of ideas merged together that create a tension both in listening and with respect to performing the piece. Additionally, this analysis of *Herma* reminds me of Henry Flynt's notion of *structure art* as opposed to *concept art* — notions both developed the same year as *Herma*, in 1961. The composer, artist and philosopher criticized the idea of music as:

> sounds made to operate as vehicles for maintaining and communicating an abstract "structure," irreducible to the sensible presence of the sounds. [As a result, *structure art* would imply] the underdevelopment of both the structure and of the sensible content of the art. Insofar as music seeks to satisfy both poles (structural and sensible/musical), at the same time, both are diluted: structure becomes impoverished as it needs to be incarnated in sounds, and music becomes impoverished to the extent that it obeys a logic external to sounds themselves.[68]

Like Flynt, and as I mentioned earlier, I do believe that the insights gained from listening to music cannot be equated with mathematical or inference-based knowledge. The radicalness of Flynt's perspective is very refreshing in that regard. Yet I disagree with *structure art* being necessarily "impoverished." Moreover, Flynt's critique of *structure art* focuses the problem on the music or the art and what *it* seeks. However, it seems more relevant to me to say that the intentions of a composer are problematic when *they* seek to satisfy both the polarities of 'the structural' and 'the sensible'. Also, Flynt's view overlooks that a composer who seeks to satisfy one of these two poles can end up incidentally satisfying the other. Pieces of music affiliated with *concept art,* and dealing primarily with the sensible presence of sounds, may be analyzed as mathematical structures (and vice versa). Examples like John Cage's *Number Pieces,* which is another *indeterminate piece,* is just one such telling illustration.

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[68] This is a "politically correct" summary of Flynt's provocative "Essay: Concept Art" found on: J-.P. Caron, "On Constitutive Dissociations as a Means of World-Unmaking: Henry Flynt and Generative Aesthetics Redefined," e-flux Journal, Issue #115 February 2021. See also Henry Flynt, "Essay: Concept Art," 1961, HenryFlynt, http://www.henryflynt.org/aesthetics/conart.html "*Contemporary structure artists… tend to claim the kind of cognitive value for their art that conventional contemporary mathematicians claim for mathematics. Modern examples of structure art are the fugue and total serial music […]* These examples illustrate the important division of structure art into two kinds according to how the structure is appreciated. In the case of a fugue, one is *aware of its structure in listening to it; one imposes "relationships," a categorization* (hopefully that intended by the composer) on the sounds while listening to them, that is, has an "*(associated) artistic structure experience*" In the case of total serial music, the structure is such that this cannot be done; one just has to read an "analysis" of the music, definition of the relationships. Now there are two things wrong with structure art. First, its *cognitive pretensions are utterly wrong*. Secondly, by *trying to be music or whatever (which has nothing to do with knowledge), and knowledge represented by structure, structure art both fails,* is completely boring, as music, and doesn't begin to explore the aesthetic possibilities structure can have when freed from trying to be music or whatever. " My emphasis in italics.

4.2. John Cage's *Number Pieces* & Computational analysis

In the previous section, I focused on the listening experience of *Herma* in relation to its musical concepts and compositional methods. I will now focus on Alexandre Popoff's recent computational methods employed to analyze Cage's *Number Pieces,*[69] mainly drawn from stochastics and statistics. I will examine how these methods reframe Cage's notion of "indeterminacy" and propose that Popoff's analysis may be seen as an act of interpretation or even performance of Cage's piece.

In the *Number Pieces,* Cage expanded his idea of "indeterminacy" by using chance operations as compositional tools specifically for building a notion of harmony and resulting into a certain unknowability for the performer and the listener. Yet, the recent computational methods of analysis of Cage's *Number Pieces* tend to make this unknowability relative, since they delineate the predictable lines in the pieces. By doing so, the analysis also highlights in my opinion the crucial role of Cage in composing the piece.

4.2.1. General presentation about the piece and Cage's notion of harmony

John Cage's *Number Pieces* are the latest cycle of pieces written by the composer between 1987 and 1992. The main feature of the *Number Pieces*' is their "time-bracket" structure. Within these time-brackets, specified pitch intervals and silence can resound without their exact beginning and end times being precisely defined in the score (See Fig 4.2).

Fig.4.2: Example of time-bracket notation in Cage's *Number Pieces*

[69] Alexandre Popoff, "John Cage's Number Pieces as Stochastic Processes: a Large-Scale Analysis." In *arXiv: Physics and Society* (2013). See also Appendix 3.2.

This technique echoes Cage's notion of *indeterminacy* in relation to the performance of a piece.[70] In the *Number Pieces,* the composer used this chance procedure to obtain a myriad of pitch combinations through the numerous possible realizations of one single score. This arguably illustrates a reconciliation between Cage and the parameter of harmony. It is an approach that combines Cage's life-long wish for harmony to be liberated from its classical, functional Western rules,[71] while also reinforcing his interest in composing simultaneous pitched tones. The latter was a preoccupation he had towards the end of his career, and at that time, Cage considered harmony very broadly, regarding it as "several sounds...being noticed at the same time, hmm? It's quite impossible not to have harmony, hmm?"[72] In Cage's music, harmony is thus understood as both the production and perception of simultaneous sounds — a performative act from the side of both musicians and listeners.

This specific notion of harmony is present in the *Number Pieces*. The chance operations applied to harmony in the piece prompt introspective states for listeners: the perception of simultaneous sounds, without no any real grasping sequences of higher-level unity. In that sense, the *Number Pieces* do not explicitly offer formal or temporal grounds or a temporal structure which might guide the listener through the unfolding of the piece. Rather, the treatment of harmony in the *Number Pieces* renders the listener oblivious to comprehending any kind of structure or musical form— and the latter specifically becomes dissolved into the perception of simultaneous sounds.

4.2.2. Computational analysis: Number Pieces as stochastic processes

Computing numerous simulations of possible performances of such *indeterminate* pieces allows for more clarity on the musical structures of these pieces which seem unidentifiable at first. This is exemplified in the work of the musicologist Alexandre Popoff, who thanks to computational methods, studied the structure of Cage's *Number Pieces* extensively. He described and analyzed *Four* and *Five* (1988) *as stochastic processes* in his 2013 paper titled 'John Cage's Number Pieces as Stochastic Processes: a Large-Scale Analysis.' Popoff's research consists in transforming the *Number Pieces* into a mathematical structure using combinatorics and computable stochastic

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[70] See John Cage, *Silence, Lectures and Writings by John Cage, 50th Anniversary Edition,* Wesleyan University Press, Middletown, 2012, p.35 and the concept of an "indeterminate composition in relation to its performance."

[71] See John Cage, *45' For a Speaker:* "Harmony, so-called, is a forced abstract vertical relation which blots out the spontaneous transmitting nature of each of the sounds forced into it. It is artificial and unrealistic" in *Silence, Lectures and Writings by John Cage, 50th Anniversary Edition,* Wesleyan University Press, Middletown, 2012, p.152.

[72] John Cage and Joan Retallack, *Musicage: Cage Muses on Words, Art, Music. John Cage in conversation with Joan Retallack*, ed. Joan Retallack, Hanover, NH: University Press of New England, Wesleyan University Press, 1996, p. 108.

processes. He states that, "[b]y averaging over a large number of realizations (which is achieved through a computer program running the determination of the parts repeatedly) we can access the probability distributions of each pitch-class set over time, thus turning the *Number Pieces* into stochastic processes."[73] Hence, thanks to computational methods, the musicologist can now turn possibilities (multiple realizations of a musical concept) into probability and probability into mathematical concepts and a logical construct. While this chain of events is nothing new, the methods for analyzing the music are.

4.2.2.1. Computational methods of analysis

The way Popoff's reduced the *Number Pieces*'s structure is important to review. He worked with the following main axioms to analyze Cage's *Five* and *Four*:

- First, he divided each piece into different *sections*, and treated each section independently.
- Popoff then defined a *set of variables* specific to each section. These variables are the pitch-class sets[74] (including silence) and their starting/ending times within their corresponding time-brackets. This step is crucial in turning a *Number Piece* score into a computable function.
- The musicologist used *Gaussian distributions* for selecting the starting and ending times in the corresponding intervals of each time-bracket — which is, in passing, a very subjective and disputable choice.
- The next step of the analysis corresponds to the simulation of numerous realizations of the piece (between 10^4 and 10^5). It consisted in the generation of *collections of random variables indexed over time*, with values in the possible pitch-class sets within each time-bracket. This means that the outer limits of the time-brackets are chosen through a random selection within the given intervals and that the selection of multiple pitches inside a time-bracket is made through a succession of random time-mark choices.
- Finally, Popoff went through the *computation and quantification of the probabilities of occurrence of the chords* in each time bracket, and eventually of *a model for pitch-class set evolution,* with the most probable transitions between pitch-class sets for an entire section of a piece and the whole piece.

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[73] Popoff, Alexandre, "John Cage's Number Pieces as Stochastic Processes: a Large-Scale Analysis." In *arXiv: Physics and Society* (2013).

[74] Popoff uses Allen Forte's notation found in *The structure of atonal music,* Yale University Press, 1977.

4.2.2.2. Uncovering of *Number Pieces*'s harmonic biases

Popoff's study allows for visualizing *Four* and *Five* of the *Number Pieces* as structures, and this enhances what could be considered as their harmonic/time biases. Instead of a purely uniform distribution of pitch-class sets, the latter were probabilistically weighted in *Four* and *Five,* some being more prevalent than others.[75] Moreover, the probability distributions of occurrence of the pitch-class sets are not fixed but vary over time. These distributions can be either uniform within a given duration among different pitch classes; or introduce different weights, some of them significantly high for specific pitch-class sets and low for others within another time-bracket.[76]

In brief, Popoff's rigorous and detailed analysis allows to redefine Cage's musical concept of harmony in the *Number Pieces* through the lens as mathematical concepts. This was an important manner of analysis for me and one that further blurred Flynt's limited distinction between structure art and concept art. Once articulated into mathematical concepts, Cage's way of thinking about harmony can be understood through a more subtle perspective — to the point where Cage's statement on harmony may seem simplistic in relation to the *Number Pieces*. Harmony is not equivalent to performing or listening to 'any simultaneous sounds.' Rather, harmony is restated as a limited set of pitches woven with rather specific intervallic relationships, and also as a controlled density of possible simultaneous sounds, carefully carved throughout a piece. As Popoff gives a clear image of the probabilities of occurrence of pitch-class sets and their evolutions throughout the unfolding of the *Number Pieces,* his study shows the predictable aspects of *Four* and *Five* — nuancing the idea of the pieces's endlessly "varied" realizations, and fundamentally displaces the usual point of focus that most of us have concerning Cage's *Number Pieces*. It also reveals the tactics of attention that Cage employed in the piece: how he carried the listeners without them noticing that their attention is directed in a rather specific way. Although Popoff explicitly mentions his study as *not* being perceptually motivated, imagining/studying the potential perceptual effects of *Four* and *Five* on listeners, thanks to their harmonic profiles, could be another interesting avenue for further understanding the piece.

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[75] For example, in *Five* Popoff studies confirm the prevalence of pitch-class sets 3-7 ([0, 2, 5]), 3-2 ([0, 1, 4]), 3-10 ([0, 3, 6]) and 4-27 ([0,2,5,8]) in Forte notation, corresponding respectively to the presence of [(M2 or m7 + P4 or P5)], [(m2 or M7 + m3 or M6)], [(m3 or M6 + A4 or d5)] and [(M2 or m7 + P4 or P5 + m7 or M2)]. In brief, the intervals that have the most chance to occur are major seconds/minor sevenths, minor thirds/major sixth, major sevenths/minor seconds, perfect fourths/perfect fifths.

[76] See Popoff's specific analysis of the first time-bracket in *Five,* ibid., pp.13-19.

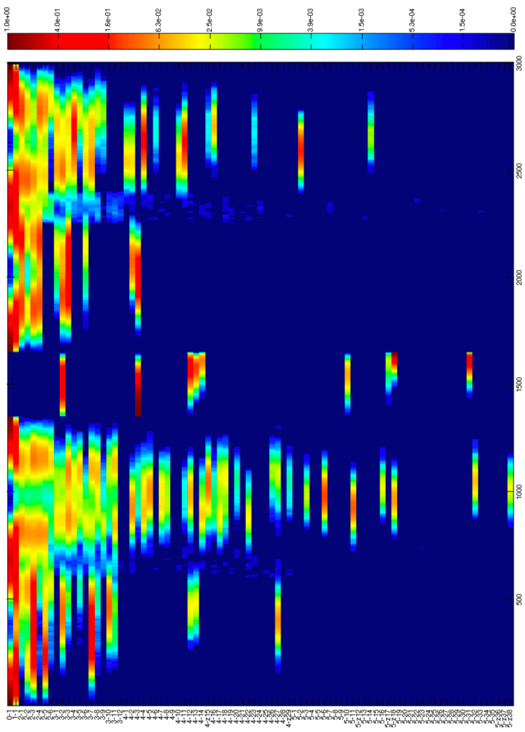

Figure 4: Heatmap of the probabilities $Pr(PCS_t = i)$ over the 87 possible pitch-class sets (in ordinate) at each time t (in abscissa) in *Five*. The colorbar indicates the corresponding probabilities in pseudo-logarithmic scale (see text).

Fig 4.3: Popoff's heatmap of the probabilities over the 87 possible pitch class sets (in ordinate) at each time *t* in Five.

4.2.2.3. Computational analysis as interpretation/performance

Popoff's analysis puts aside the (essential) reality faced by musicians when playing the piece and listening to one another, especially the assumption that the entrance and exit times should follow a Gaussian Curve. This view on the performance of the piece is actually restrictive and biased. But because of it, the paper raises interesting questions about the notions of interpretation and performance, namely, I view Popoff becoming a kind of interpreter/performer of Cage's score. I would also claim that his approach in many ways resembles my own analysis of *Arbor Vitae*. To me, this illustrates how the boundaries between musical analysis and performance can become blurred through the use of computers. And in fact, I found out much later that Popoff started a "365-days projects" in 2020 where he posted each day an algorithmically generated version of John Cage's Five on Youtube, based on his algorithmic reconstruction of the piece.[77] Despite stopping his posting after 47 days, the format indicates Popoff's position on the piece was not solely analytical anymore but also very performative and artistic.

[77] Alexandre Popoff, *John Cage, Five - A 365 days project,* Alpof, September 2020, https://alpof.wordpress.com/2020/09/05/john-cages-five-a-365-days-project/. See also Alexandre Popoff, The Number Pieces of John Cage (7), Alpof, January 2017, https://alpof.wordpress.com/2017/01/13/the-number-pieces-of-john-cage-7/.

Finally, Popoff's studies indirectly enhance how musically and computationally efficient and elegant his compositional model is — even if that was not Cage's primary intention. The *Number Pieces* provide complex output based on simple instructions, one being an ideal situation for computability theorists and computer scientists. I discovered only much later that some of the *Number Pieces* were partially computed thanks to Andrew Culver's program *musicfor*, which generates time brackets pitches, dynamics, and "specials".[78] As much as I do not have enough information about how this program was used by Cage, this anecdote confirms that the general consensus about Cage's music —including my own— still needs to be refreshed.

Last but not least, rather than prompting me to create my own algorithmic reconstruction of the piece,[79] Popoff's paper has certainly inspired some new musical avenues that I could explore in the future. For example, I consider engaging with compositional strategies similar to the *Number Pieces* but starting from programming and thus from an awareness of the structure of the piece — one that I could share with the instrumentalists. This composition could combine:

- a Cagean *indeterminate,* aleatoric dimension in relation to its performance, allowing for performers to work freely and independently from each other;
- and a rigorous structure, fundamentally be based on algorithmic, probabilistic processes. If this structure would be known by the performers, I could imagine it may not only inform their way of performing but also create a sense of collective cohesion and understanding.

Overall, I believe the above points are compositional interests that I am already exploring, although in an embryonic form (as solo pieces). Presently, a future aim of mine would be to expand on these ideas further with several instrumentalists.

[78] See Anarchic Harmony, *Cage Programs* https://www.anarchicharmony.org/People/Culver/CagePrograms.html.

[79] This has been done by Popoff and also Benny Sluchin, Mikhail Malt, *A computer aided interpretation interface for John Cage's Number Piece Two* Journées d'Informatique Musicale (JIM 2012),, 2012, Mons, Belgium.https://hal.archives-ouvertes.fr/hal-01580153/document.

5. Other parallel trajectories between musical and mathematical creativities

The pieces I will present in this chapter were all influential in my work as they literally "blew my mind" in different ways: through their music, their structure, their approach to harmony, their philosophical grounds, their distinct connexions to mathematics. The aim then in this section will be to illustrate further the singularities with which one composer may undertake these processes of transition between mathematical concepts, musical concepts and percepts.

5.1. Clarence Barlow, Approximating Pi (2007)[80]

It seems to me that a particular branch of computer music consists of presenting the computation of mathematical objects as a musical process. This shows another, specific take on how mathematical objects may be used for music derivation. I find two particularly illustrative examples of this are the two works *Approximating Pi* (2007) and *Approximating Omega* (2010), respectively written by the composers Clarence Barlow and Michael Winter. I will go back to *Approximating Omega* in the last section of this chapter, but as indicated in their titles, these two pieces deal with *approximation*. Computability theorists often work with *approximation,* i.e. partial computations to represent mathematical objects whose exact form or exact numerical number is unknown or difficult to obtain,[81] in this case the transfinite number Pi or the highly complex and truly random number, Omega. Barlow and Winter's pieces thus work in the realm of the 'non-computable' via partial computations, tying together musical and *approximation processes*.

The computer-based electronic music installation, *Approximating Pi,* illustrates a point of junction between just intonation, mathematics, computability theory, programming, and harmony as a perceptual phenomena. In contrast to Xenakis's *Herma* for instance, which is somewhat controversial in that regard, the structure of *Approximating Pi* is fully determined by computations, i.e. those for approximating Pi. As a reminder, Pi may be represented as the following series: π= 4(1- 1/3 +1/4 - 1/7 + 1/9…), forming a *convergent sequence* gradually approaching a limit point. The limit of this sequence is the value each term of the sequence tends towards, indicating when an irrational number such as Pi finds a form of stability where it can be *approximated*. In that case, Pi converges towards 3.1415926535. In *Approximating Pi*, Barlow proceeds to the computation of this series *as* a sample-based synthesis procedure where the "amplitudes of ten partials of a complex

[80] Clarence Barlow, *Approximating Pi,* Compositions, http://clarlow.org/compositions-by-year/approximating-pi-8ch15/ and Akshay1992, "ApproximatingPi", Github, https://github.com/akshay1992/ApproximatingPi. See Appendix 3.3.

[81] Wikipedia, "Approximation" https://en.wikipedia.org/wiki/Approximation.

tone would reflect the first ten digits of the progressive stages of convergence [of Pi] – with each added component of the series – toward the final value."[82] This mathematical and musical process gradually reconstructs a stable saw tooth spectrum containing a set of ten partials of an overtone series.[83]

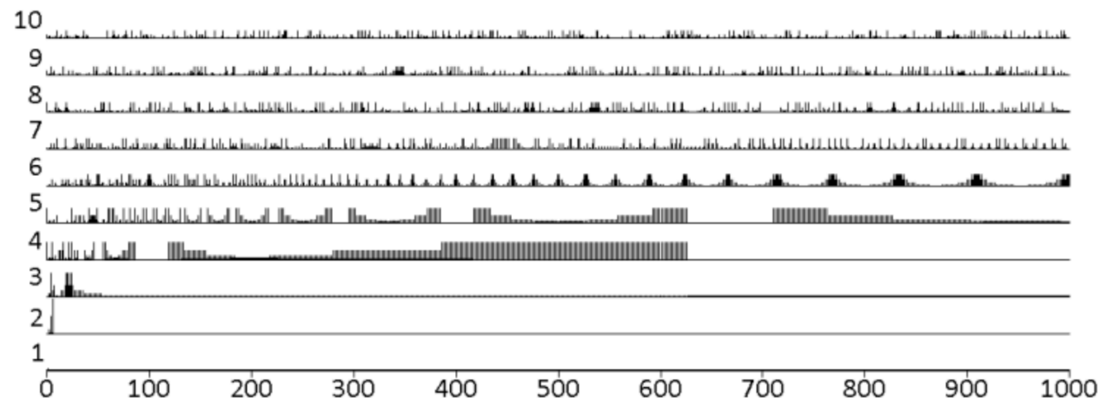

Fig 5.1: The digits of the first 1000 approximations of π, shown to the tenth digit, as amplitudes of a 10-partial spectrum in Barlow's *Approximating Pi*.[84]

As a listener, a process of "convergence" towards a tonal stability is made very transparent. The listener is first invited to follow the timbral complexity of the wave forms, their construction by means of a gradual timbral stabilization, and this allows them to also gradually attune to the harmonic relationships present in the piece.

A very different illustration of a composer's relation to mathematics from the ones evoked until now is found in Catherine Christer Hennix's work.

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[82] Clarence Barlow, "Algorithmic Composition, illustrated by my own work: A review of the period 1971-2008" in *Proceedings of Korean Electro-Acoustic Music Society's 2011 Annual Conference (KEAMSAC2011)*, Seoul, Korea, 22-23 October 2011, p.8.

[83] ibid.: The piece was realized wholly in Linux GNU Pascal. Barlow "decided on a time frame of 5040 samples, i.e. 83⁄4 frames per second. All time frames contain a set of ten partials of an overtone series, multiples of the frequency 83⁄4 Hz, which automatically results from the width of the frame, 5040 samples."

[84] ibid.

5.2. Catherine Christer Hennix and Brouwerian continuum

One could (and should) dedicate an entire research project exclusively to studying the Norwegian composer, musician, poet, artist, mathematician, philosopher, Catherine Christer Hennix. Her work results from an assemblage of various artistic practices (jazz, raga, maqam, blues, poetry, *noh* theater), and teachings (mathematics, logic, buddhism, muskim sufism) — and this is probably only the top of a much broader iceberg. For me, her body of works represents the epitome of "transdisciplinarism," a highly personal artistic practice and one that is also involved with political and social engagements of the times. If there is little, proper documentation and research about her work, it is still worth mentioning.[85] In general, Hennix's art may be described as attempts to wed mathematical formalism and introspective states of mind.[86] Logic and numbers are not only tools to model music in Hennix's works; rather, they point at a much broader "mathematization" of art-making and way of being. In brief, Hennix's relation to mathematics is existential, and this is especially shown in her music.

5.2.1. Catherine Christer Hennix & intuitionism

Hennix's work is permeated by so-called *intuitionism*, a radically novel —and controversial — conception of mathematics and mathematical creativity consisting of a *mathematical theory of introspection,* founded by the mathematician Luitzen Egbertus Jan Brouwer around the early 1900s. As a philosophical position, intuitionism puts the "creative subject" and its psyche at the centre of the mathematical activity, where it can be understood as the fruit of internal acts of thinking. Because reasoning is always attached to a specific mathematical context and the creative subject, intuitionists acknowledge mathematical notions have only a provisional, relative existence. In Hennix's music, the role of the composer and performer are inspired from the Brouwerian "creative subject," and this emphasises putting one's intuitions at the center of a rigorous creative process. Thus similar to the Brouwerian mathematical activity, the musical activity, is interpreted as the result of internal or introspective states of mind; it is highly subjective, revisable, and offers transitory solutions to a musical inquiry.

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[85] In general, her work is not very well known or documented and this is partially due to Hennix's indifference to releasing her music in conventional recording formats.

[86] Spencer. Gerhardt, "Domains of Variation: Choice Sequences, Continuously Variable Sets, Remarks on the Yellow Book." In *Blank Forms Journal, Vol. 4: Intelligent Life,* Blank Forms Editions, 2019, p. 102.

5.2.2. Engaging with formalizations of the continuum

If Barlow's *Approximating Pi* had to do with the mathematical formalization of irrational numbers, Hennix's music mostly engages with mathematical formalizations of the *continuum,* i.e. "theories or models that explain gradual transitions from one condition to another without abrupt changes or discontinuities."[87] This engagement was nourished by two intermingled sources.

5.2.2.1. Continuum and Raga: "attunement" to musical intervals

Hennix's interest in formalizing continuous phenomena and infinity in her music stems from her encounter with drone music and Indian *raga* — a melodic framework for improvisation contained as a substructure within the tambura sound.[88] In an interview, Hennix explains that "You get your first intuitive acquaintance with infinity through the raga and then mathematics amplifies this concept of infinity by teaching you to formally manipulate it on paper with symbols."[89] Hennix's musicianship results from a process of first *attuning* to the perception of justly tuned harmonic relationships and/or partials over a fixed fundamental, sometimes intermingled in a complex sonic event, like the tambura drone. On this basis, Hennix then formalizes her musical compositions, ones that are endebted to mathematics. I can relate to this approach, as seen in my attempts to get "attuned" to *Arbor Vitae* or the tuning systems I work with, and how my pieces are also developed on the basis of simple mathematical concepts.

Attunement is radically put in the forefront of Hennix's work. Rather than fixed notated pieces, Hennix's music evolves through a given harmonic framework, reminding of La Monte Young way of considering composition as:

> a mode of generation, always developing in time through the performer's attention. Intervals are not treated as relations of completed points in musical space, but rather unfinished sequences of observation, subject to further refinement.[90]

This is shown in the way Hennix systematically and rigorously improvises with acoustic instruments through their acoustical properties, or self-made justly tuned synthesizers like in her

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[87] Wikipedia, "Continuum", https://en.wikipedia.org/wiki/Continuum. This is the most basic definition I found to avoid getting too deeply into philosophical/mathematical "abysses" right away!

[88] Definition inspired from Hennix's own definition in Marcus Boon, "Basically One to Infinity: an Interview with Catherine Christer Hennix." *Blank Forms Journal, Vol. 2: Music From the World Tomorrow,* Blank Forms Editions. New York, 2018, pp.121–142.

[89] Marcus Boon, "Basically One to Infinity: an Interview with Catherine Christer Hennix", ibid.

[90] Gerhardt Spencer, "Minimalism and Foundations," ibid. p.236.

early *Electronic Harpsichord*.[91] Moreover, ***at****tunement* is what is primarily present in her work and shared with the listeners, developing in time according to their attention. So despite Hennix's works owing their origin to inference and logic, they propose an epistemological perspective that is fundamentally apperceptive — ways of knowing exclusively based on perception.

5.2.2.2. Continuum and Brouwerian's choice sequences

When getting to know Hennix's compositional methods, one can observe one main compositional pattern, i.e. how her treatment of musical intervals is influenced by some foundational shifts or oscillations in the mathematics she works with. Spontaneously, I believe that my approach to musical intervals is similarly influenced by such an foundational oscillation in the mathematics I am inspired by (mainly, between finite and infinite probabilistic spaces).

Frequently, Hennix's compositions express a shift from classical to intuitionistic mathematics, or from constant structures to variable ones.[92] Mainly, Hennix's music and ways of working with musical intervals are partially inspired by set theory but mainly by the Brouwer's notion of continuum. Based on a disapproval of the classical, set-theoretic, reduction of the world's phenomenon to discrete terms and completed sets,[93] the Brouwerian continuum is constituted through *choice sequences*. The latter are blurred mathematical structures which are never being fully finished and dynamically emerge or develop in time according to an idealized mathematician's attention and intuition.[94] More precisely, these sequences are constructed from a finite set of elements, where each element is successively and freely chosen by a creating subject in a potentially infinite process. *Choice sequences* "may follow pre-ordained rules (law-like sequences), while others are generated quite freely by the subject (a lawless sequence),"[95] meaning that they are in fact random.

Delving into choice sequences is beyond the scope of this thesis but one can intuit how they shaped Hennix's work. The latter evolve between rigorous modal musics (law-like sequences), her

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[91] The AmbientFox, *Catherine Christer Hennix - The Electric Harpsichord*, https://youtu.be/eXxobmct4xY . See Appendix 3.4.

[92] Gerhardt Spencer, "Domains of Variation: Choice Sequences, Continuously Variable Sets, Remarks on the Yellow Book", ibid p.130.

[93] Set theory provides an understanding of the world mainly in discrete or disconnected terms, and primarily tied to constant structures. In this context, the mathematical continuum is more of an additional structure added upon a pre-existing discrete collection of objects; and mathematical objects in general are seen as objective entities ruled by logic and deduction, independent of the thinking mathematician,.

[94] Gerhardt Spencer, "Domains of Variation: Choice Sequences, Continuously Variable Sets, Remarks on the Yellow Book", ibid p.114.

[95] Gerhardt Spencer, "Minimalism and Foundations" in *Simplicity. Ideals of Practice in Mathematics and the Arts,* New York: Springer, 2017, p.235.

own compositional decisions and/or improvisations with other musicians (lawless sequences). Often, new musical modes emerge in Hennix's music by the way she layers together just intonation modalities from traditional, rigorous modal musics (raga, maqam, blues) in one piece, through improvisation. Additionally, Hennix's compositions are developed thanks to computer generated processes. In fact, Hennix treats a computer as an idealized mathematician with perfect memory and indefinite attention, which can proceed with "infinitely long spreads of musical events, locked together by some appropriate algorithm that recursively generates each new step on the basis of the preceding ones."[96] The latter quote highlights the importance of computable and recursive functions in Hennix's compositional process, functions which were introduced multiple times throughout this thesis as well.

For me, some aspects of Hennix's radical approach to mathematics resonate with the work of Michael Winter, which will illustrate a last approach in this chapter to the process of translating mathematical objects into musical structure.

5.3. Michael Winter, *Approximating Omega* (2010)[97]

Michael Winter's *Approximating Omega* synthesizes some of the ideas found in Barlow and Hennix's work. Winter derives computer-aided and open scores in the form of 'harmonic framework' for the performers, from the very specificities of a mathematical object, *Omega*.

5.3.1. Influence of digital philosophy

A radical aspect of the work of Winter's is that it finds its source in digital philosophy, and the hypothesis/basis where everything in life, any phenomena, any experience can be theoretically computable. Admittedly, Winter is highly influenced by Algorithmic Information Theory (AIT), founded by the mathematician Gregory Chaitin, whose playfulness with mathematics is reminiscent of Brouwerian's intuitionism and thus an approach to mathematics that foregrounds introspective practices. For example, Chaitin equates the limits of reasoning with the limits of computability and calls a mathematician trying to define randomness as "a rational mind trying to find its own

[96] Spencer Gerhardt, "Minimalism and Foundations," ibid. p.235.

[97] Listen to the piece here: Muirgene Leonore Gourgues, *Approximating Omega,* Edition Wandelweiser Records, 2019 https://soundcloud.com/muirgeneleonoregourgues/approximating-omega and an excerpt. See Appendix 3.5.

limits."[98] Chaitin also presents a very different mathematical paradigm whose foundations are randomness and probabilities. For him, AIT implies:

> that some mathematical truths are fundamentally probabilistic, that *there is randomness in the foundations of mathematics*, that the truth content of some statements in pure mathematics is grey, not black or white, that sometimes truth is probabilistic, not sharp and clear […] AIT shows that randomness and probabilities arise naturally in pure mathematics[…].[99]

Chaitin goes even further by saying that "the reason why mathematicians are changing their working habits"[100] is due to the computer, which has turned mathematics into a far more "experimental"[101] science, one that is gradually imitating methods employed in physics.

Winter explains that in his music he is not interested in demonstrating a mathematical concept to the listeners. At the same time, his music is also not exclusively based on the mere perception of sounds, but lies somewhere in-between. This is expressed in the first part of *Approximating Omega,* which is an optional part of the piece and may be accompanied by a reading of texts by Chaitin (to give more context about the mathematical aspect of the work). A major takeaway from this for me was that a possible didactic dimension of the work remains optional.

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[98] Gregory Chaitin, *Exploring Randomness,* Springer, 2001, p.162.

[99] ibid.

[100] ibid.

[101] ibid.

5.3.2. Deriving a piece's structure from the structure of a computer program

The second, and non-optional, part of the piece is fully derived from the structure of the computer program that approximates the truly random, highly complex number Omega — found in the work of Chaitin. Omega is the probability for a computer program to halt. In its essence, Omega is random, and its randomness challenges the limits of computability. Yet, as a value it is able to be computed thanks to a very sophisticated computer program with multiple nested routines — akin to 20-piled matryoshka dolls. As a comparison, *Arbor Vitae*'s *nestedness*, which was already fascinating and challenging for me to understand, had less than five such nested routines (which were themselves obviously much simpler mathematical processes).

```
(('(&(V)(('(&(A)(('(&(R)(('(&(W)(W('(O)))))('(&(n)(*0(*.(R(Vn())n))))))))('(&(xy)(/(.x)(/(.y)()(*0(Rx(-
y))))(^(R(-x)(y))(*(+x)())))))))))('(&(xyz)(/(.x)(/(.y)(/z('(1))())(A('(0))yz))(/(.y)(Ax('(0))z)(*(=(+x)
(=(+y)z))(A(-x)(-y)(/(+x)(/(+y)1z)(/(+y)z0)))))))))))('(&(xy)(/(.x)(/(.(+(?n('(!(%)))y)))()('(1)))(A(V(
-x)(*0y))(V(-x)(*1y))0)))))

Above is the program approximating Omega given in its ascii representation. Note that the symbol O
represents a list of 1s with a length that determines how many bits of the binary expansion of Omega
are approximated.
```

Fig. 5.2: Computer program approximating Omega in its ascii representation. Michael Winter, *Approximating Omega*, score (p.5)

In contrast to Barlow's *Approximating Pi,* the piece is not, as such, about the mathematical object *Omega* but rather the *nestedness* of the routines present in the computer program approximating *Omega*.[102] In that regard, this part of the piece illustrates a very rigorous translation *symbol-by-symbol* from the computer program's structure (see Fig. 5.2), into sonic events: distinct (or primarily short) sounds representing each symbol and the entrances and exits of various continuous sounds that demarcate the expressions in the program. Furthermore, Winter attempts to capture the "phenomenon" of realizing this incomputable mathematical object into music.

> Apart from a structure borrowed from the program itself, the transcription of a computer program that approximates a maximally complex number such as Omega is perhaps the most pure realization of such a mathematical object since any computation or subset does not so fully encapsulate the phenomenon.[103]

[102] Michael Winter, *Approximating Omega,* unboundedpress, 2010.

[103] ibid.

Assign each part played by a pitched instrument a distinct pitch class from the following:
(D+0, A+2, F#-14, C-31, G#-49, A#+41, D#+5, F-2, G#+28, C+30, C#+45, F-49, F#+29, G+12, A-34, B-26, C#-41)

Option 1 (start time-unit, end-time unit):

Part 1	Part 2	Part 3	Part 4	Part 5	Part 6	Part 7	Part 8	Part 9	Part 10	Part 11	Part 12	Part 13	Part 14	Part 15	Part 16	Part 17
0,336	1,259	3,258	5,7	9,136	11,135	13,15	17,72	19,71	21,23	25,42	27,41	29,31	34,39	36,38	58,63	61,62
	260,335	262,334	8,257	137,256	139,255	16,134	73,133	75,132	24,70	43,69	45,68	32,40	53,66	56,65		
			264,267	270,273	276,295	141,145	148,151	154,157	77,80	83,86	89,92	47,49	101,104			
			268,333	274,304	296,297	146,254	152,181	158,169	81,131	87,107	93,94	50,67				
				305,332	298,303	278,294	182,253	170,180	161,166	108,130	95,106	98,105				
					307,318	300,302	280,293	184,187	167,168	163,165	110,120	112,115				
					319,330	309,312		188,198	172,177	174,176	121,129	116,119				
						313,317		199,252	191,196	193,195	209,212	123,126				
						321,324		283,291	201,215	203,206	228,231	127,128				
						325,329			216,251	207,214	232,240	234,237				
									285,290	218,221	241,249	243,246				
										222,225						
										226,250						
										287,289						

Appendix
Sustained Tones (start time-unit, end-time unit):
((0,336),(1,259),(3,258),(5,7),(8,257),(9,136),(11,135),(13,15),(16,134),(17,72),(19,71),(21,23),
(24,70),(25,42),(27,41),(29,31),(32,40),(34,39),(36,38),(43,69),(45,68),(47,49),(50,67),(53,66),
(56,65),(58,63),(61,62),(73,133),(75,132),(77,80),(81,131),(83,86),(87,107),(89,92),(93,94),(95,106),
(98,105),(101,104),(108,130),(110,120),(112,115),(116,119),(121,129),(123,126),(127,128),(137,256),
(139,255),(141,145),(146,254),(148,151),(152,181),(154,157),(158,169),(161,166),(163,165),(167,168),
(170,180),(172,177),(174,176),(182,253),(184,187),(188,198),(191,196),(193,195),(199,252),(201,215),
(203,206),(207,214),(209,212),(216,251),(218,221),(222,225),(226,250),(228,231),(232,240),(234,237),
(241,249),(243,246),(260,335),(262,334),(264,267),(268,333),(270,273),(274,304),(276,295),(278,294),
(280,293),(283,291),(285,290),(287,289),(296,297),(298,303),(300,302),(305,332),(307,318),(309,312),
(313,317),(319,330),(321,324),(325,329));

Fig. 5.3: Michael Winter, excerpt of instructions of *Approximating Omega* (p.6)

The above instructions in Fig.5.3 consist of directions similar to a Cagean open form score, exclusively providing pitch classes and start and end times, depending on the structure of the computer program. It is important to note that the more nested the computer program, the denser the sounding of the piece. The piece puts an emphasis on harmony as a perceptual phenomenon, basing it on rational intonation and sustained tones. Thus to me, this harmonic framework is again slightly reminiscent of Hennix work and, more hypothetically, Brouwerian's choice sequences.

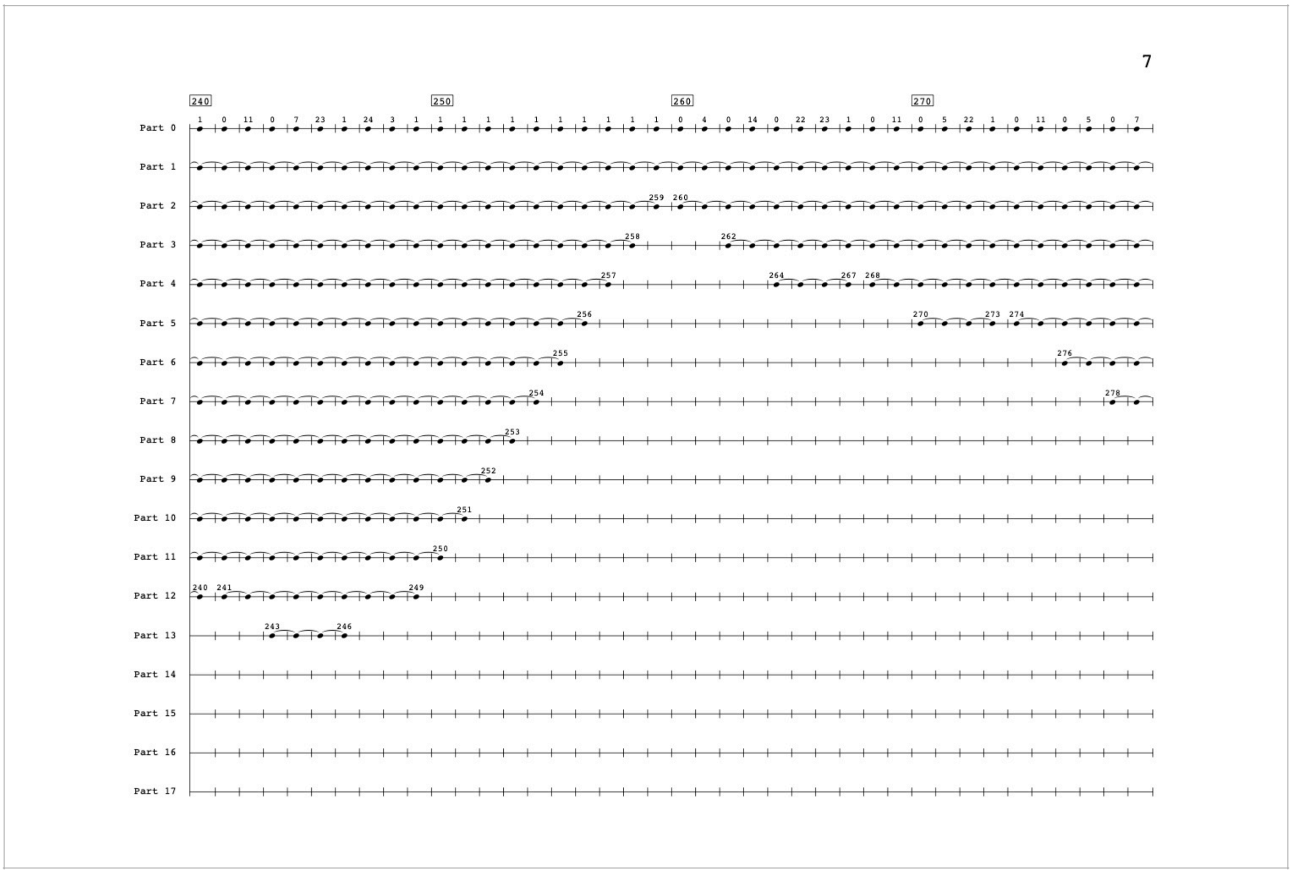

Fig 5.4: Michael Winter, excerpt of the score of *Approximating Omega* (p.7)

I believe *Approximating Omega* is a unique sort of computer music where incomputability plays a central role in its *raison d'être*. It also derives music from the nesting of routines of a computer program — a reversed approach to nestedness than the one found in *Arbor Vitae,* where nested routines resulted from imagining how the music could sound.

In summary, all these ideas are very inspiring to my work and offer a broader perspective on how and what 'computer music' or 'mathematically driven music' can be like, that being the subtleties and gradations between deterministic and non-deterministic compositional strategies, both in computer-generated sonic processes or for realizing scores for performers.

6. *Arbor Vitae,* sonification

My analysis immersed me in *Arbor Vitae*'s algorithm and this was a significant experiment, one that forced me to take a closer look at the "gradations and the back-and-forth of the concepts" used in the piece. Moreover, this allowed me to see how the emergence of 'musico-mathematical' thought can be observed.[104] What I find captivating in the piece is that Tenney goes well beyond an initial set theoretical construction for pitch; instead he built a much broader musical edifice from elements and configurations of diverse complexity and mathematical disciplines (mainly set theory, stochastics, probabilities). In 2022, I finally sonified my take on the piece's algorithm using sine tones,[105] which was another meaningful experience in different ways and doing this greatly informed, *for blandine,* my last piece composed for this research project.

6.1. Two distinct aesthetic experiences

In contrast to the string quartet version, which I see as an "embellished" version of the piece's structure, enveloped by the timbre of the string instruments and expressive changes in dynamics, I have always believed I could listen to another version of the piece, one reduced to its "bare skeleton." In other words, I have always wanted to bypass the timbral complexity in the piece and get a more direct access to the harmonic relationships and tuning, unaffected by the mass of sound characteristic of string quartets. The synthesized version is and sounds much more precise and this confirmed my previous experiences in *for edgars,* that being, an extremely "clean" timbre allows for more precision in performing and grasping harmonic relationships.

Additionally, listening to the two versions of *Arbor Vitae successively* brought to light some of my other compositional ideas. For example, I started reflecting on the possibility of emphasizing the distinctness between an instrumental part and an electronic part in a piece, while suggesting—through listening—how the two sound layers stem from the same structural and conceptual grounds.

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[104] Paraphrasing Fernando Zalamea's expression in *Introduction* to Lautman Albert, *Mathematics, Ideas and the Physical Real,* Continuum International Publishing Group, 2011, pp.xxiv- xxvi.

[105] See Appendix 1.3.

6.2.A form of transcription & interpretation of the piece

Similar to what I noticed with Popoff's study and 365-days act with Cage's *Number Pieces,* I found out that reconstructing the piece was a form of transcription and interpretation of the piece's original algorithm. The transcription part is easily understood: as I did not have access to the original program of the piece, I just transcribed it to a different programming language. The process of coding a precise, and as close a reconstruction of the piece as possible, was paved with uncertainty and some overall ambiguity. In reality, I occasionally had to take certain "interpretative" decisions about the algorithm of *Arbor Vitae*. These decisions had an impact on the output of my reconstructed algorithm, and later, its sonification. For instance, my sonification ignored dynamic changes. Certainly, I felt inspired by the generative aspect of this reconstruction, and the ease with which one can realize and listen to many more renderings of the same piece (because it is based on probabilistic processes). In short, I would like to expand on these ideas for future works, specifically, I would be curious to try running the algorithm with a different synthesis, a different tuning system, a different length and overall pacing, etc. In other words: re-interpreting the piece's musical concept.

6.3. Imagination

Reflecting on Tenney's compositional method—starting from algorithms and without "hearing" any sonorous renderings of them for several months—I realized how his method entailed not only risks but also a necessity to trust the eventual sounding of the piece. The piece was the last one Tenney ever wrote, mostly from the hospital with the assistance of Michael Winter. After working on the algorithm quite extensively, Tenney listened to a synthesized version of the piece with sine tones and was ecstatic. He passed away a couple of weeks after, before hearing any performance of the piece. This is not a mere anecdote to me, but crucially illustrates the composer's ultimate compositional method, combining highly degrees of formalization and imagination, of algorithmic processes and inner listening — and again, trust.

7. *for blandine and maciej* (2022)

The composition *for blandine and maciej*[106] is a piece for harpsichord and computer. The piece encapsulates many of the areas explored throughout this thesis: rational tuning systems and harmony, probabilistic processes for generating computer-based music/scores, composing/analyzing musical structures through the lens of mathematics and computable processes, and exploring the perceptual effects of a piece. Overall, working on this piece reminded me of Tenney's idea of indeterminacy as "points in a circle," which involved (in this case) revisiting some of my own compositional and listening experiences, and to circulate between different bodies of knowledge. Through this chapter, it is my hope that the reader may follow the constellation of ideas that flourished from and after writing this piece. Initially, the piece started from a similar approach to tuning and probabilities as found in *for edgars*. Gradually, the piece deviated towards another approach to the harpsichord, taking into consideration some aspects of its timbre but also its historical dimension. The latter led me to engage with a notation system inspired by L. Couperin's *Unmeasured preludes,* adding more dimensions and flexibility to the mathematical concepts from which the piece originated. The mixture or tensions between computer music and early music conventions occurs in the performance as well as in the listening of the piece. In this piece, I ended up exploring the merging of computational creativity in music with baroque music conventions; also largely due to the involvement of the very open minded and gifted harpsichordist Maciej Skrzeczkowski.

7.1. Brief description of the piece

The piece initially took shape following the recent passing away of the harpsichordist Blandine Verlet, a member of my family whose way of playing the harpsichord and interpreting music and sound were personally very influential. Differently from composing *for edgars,* which was devoid of considerations about the historical specificities of the electric guitar, I was curious to examine such historical aspects of the harpsichord, its conventional role and expressivity in early music pieces. Perhaps more than any other instrument, I am particularly fascinated by the harpsichord because of how the creation of phrasing is intimately bound up with the instrument's timbre and resonance.

[106] See Appendix 4.5.

Similar to *for edgars,* the piece combines two probabilistic spaces (finite/infinite) resulting in two different musical logics (*harmonic relevance* and *harmonic irrelevance*), that are expressed in two diametrically different acoustic and electronic sound layers.

7.1.1. Harpsichord part

I wrote for a double-manual harpsichord with the two keyboards using slightly different extended just intonation systems for each keyboard.[107] Throughout the piece, the harpsichordist transitioned from one keyboard to the other, and from high notes to lower ones. This simple trajectory allowed me to work with the full range and timbre of the instrument.[108] Rather unknowingly, I also developed a very unconventional and challenging technique (never seen by Maciej before) of simultaneously playing the two keyboards all at once. As many things in my work, this technique came from not knowing what is doable for the instrumentalist. Eventually, it brought a certain playfulness for the performer, as well as beneficial limitations for the general pacing of the piece.

7.1.2. Electronic part

The electronics of the piece consisted of 24 sine waves, initially tuned like the two keyboards of the harpsichord. In contrast to the harpsichord, the sine waves also remained in the same mid-high register. They transitioned from one type of tuning to the other thanks to real-time, probabilistic processes. In the middle of the piece, the frequencies of the sine tones were neither tuned to one tuning system, nor the other, but somewhere in-between. This introduced additional microtones other than the ones found when playing the two harpsichord keyboards together. As discussed in *for edgars,* the sine tones created auditory beatings, their frequencies randomly change *within small intervals* corresponding to the small differences between the two types of tunings (less than a semitone). Importantly, there were two groups of sine tones in the piece: some were solely sustained (group A), like in *for edgars*; while others had short attacks (group B) and were thus similar to the attacks made by the harpsichord. The decays of group B sine tones were made longer throughout the piece. By doing so, the simple effect I was after was to mirror the short

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[107] See tuning instructions in Appendix 4.1. I will not get into the details of the tunings in this Chapter. They are different from *for edgars* but follow similar principles. The difference between the two tunings is bellow a semitone. Many thanks to Richard Barrett for having suggested the idea of using two different tunings on each of the harpsichord's manuals.

[108] The 2 keyboards of the harpsichord sound differently, one being more brilliant, the other closer to a luth.

decay of the harpsichord's highest keys, and how this decay of gets longer when moving towards the lower range of the instrument.

7.1.3. Combination of the two sound layers

The combination of the harpsichord and electronic parts, either juxtaposed or sequentially presented, was the most crucial aspect of the piece. Because this combination affects the function given to the sine tones in the piece, it may also provide different points of focus for the listener.

If the two parts were played simultaneously, the sine tones would evoke the sound of the harpsichord while potentially "clashing" with it. At the beginning of the piece, the group A of sine tones evokes the short attacks/sounds of the harpsichord's highest keys. As the harpsichord part then moves towards lower registers, the 24 sine tones (group B + A) start acting as "resonators" of the harpsichord. This juxtaposition required to find a delicate balance between the limited resonance of the harpsichord strings and the sustained characteristics in the electronics. Like in *for edgars,* the harpsichord was more or less following the general timeline of the electronics, while being loosely synchronized with it.[109] As a listener, I felt that the juxtaposition of the two layers invited me to mainly focus on *an immediate perception* of harmonic relationships or the simultaneity of interactions between the two sound layers.

However, if the electronics were played after the harpsichord part, the sine tones were reminiscent of the harpsichord part and re-informed or transformed the experience of the whole piece — echoing some of my reflections and experiences with *Arbor Vitae*. In this configuration, the durations of each note and pacing on the harpsichord part remained completely up to the performer (incidentally, this way of working was more coherent in my piece, *unmesured preludes,* see Section 7.3). Then, the sine tones in *for blandine and maciej* endorsed two functions — related to how sine tones are very basic sounds (with the lowest amount of timbral information). First, the sine tones evoked a reduced representation of the harpsichord’s timbre (i.e. short attacks and short decays on the highest registers, and slightly longer in the lower register, with its resonance being maintained by sustaining notes). But what I found was that the sine tones also conveyed a reduced representation of the *piece's structure,* which consisted mostly of pitch material that accumulated as melodic units and was composed from transitions of tunings that did not change in register. In this case, as a listener, I could grasp the structural similarities between the two parts while also experiencing their distinctness. Thus this sequential presentation prompted an awareness of the

[109] I had written an open score where the performer spent 120 seconds per page. They had to get used to play for 9'00" and to stop exactly at the same time as the electronics. The freedom of the performer was conditioned by external temporal boundaries.

music's structure based on previous listening experiences and a succession of "mental impressions" — what Flynt calls *(associated) artistic structure experience*.

Eventually though, I opted for an in-between solution: the harpsichord part was played first by itself, then a sparse version of the electronics was introduced on a distant loudspeaker at a *pianissimo* dynamic during the second half of the harpsichord part. Finally, the electronics were soloed at the end of the piece, from *pianissimo* to *mezzo-piano*.

7.2. Multiple re-writings/transcription of the score

Working on this piece raised many questions related to computer-aided composition, notation, interpretation, and clashes of early music and algorithmic music. To illustrate some of these questions, I will share three versions of the same page of the score[110] (corresponding to the 3'45"- 5'40" section in the recording of the piece[111]). Each version shows a gradual discovery, both for the performer and I, of how to work with the flexible but bounded parameter of durations in the piece — an ironic outcome as I had not originally planned to explore this aspect of composition so intensely.

7.2.1. First version of the score
Bare output of SuperCollider algorithm: random sequences of pitches

The algorithm I worked with is very similar to the one in *for edgars*. Following the idea of transitioning from one tuning to the another, different sets of pitches are sequentially used for the random selection of each individual pitches. I then mainly implemented weighing functions in my algorithm. The latter gradually added or subtracted pitches throughout the piece by giving varying weights to certain pitches. In general, more available pitches for the random selection were made towards the middle of the piece, and fewer pitches can be found at the beginning and end. This had an influence on how pitches were successively repeated (or not) throughout the piece: less successive repetitions at the middle of the piece, many more at the beginning and end. Additionally, overall, the pitches were deployed first on the upper register of the harpsichord and gradually transposed down, including to the lowest notes of the instruments.

Each stave in *for blandine and maciej* corresponded to specific probability distributions. Two types of probability distribution were used: uniform distribution (allowing for each of the

[110] See Appendix 4.1 for the full score.

[111] See Appendix 4.5.

available pitches of the random selection to have an equal chance of being selected) or a distribution based on Barlow's harmonicity function. This was done to order the available pitches from simpler to more complex harmonic relation to the root note of the piece (G); and then the pitches that were harmonically simpler would have more chance to be picked by the algorithm. In practice, when using harmonicity probability distribution, the resulting random sequences of pitches conveyed a stronger presence to 'purer' intervals (octaves, fifths…) and had more cadential nature than those sequences of pitches computed with uniform distribution.

No durations were generated with my probabilistic algorithm, only sequences of pitches. Thus in the first version of the score, see Fig.7.1, note values do not represent duration. Also, the different ratios above the notes simply indicates the tuning systems.

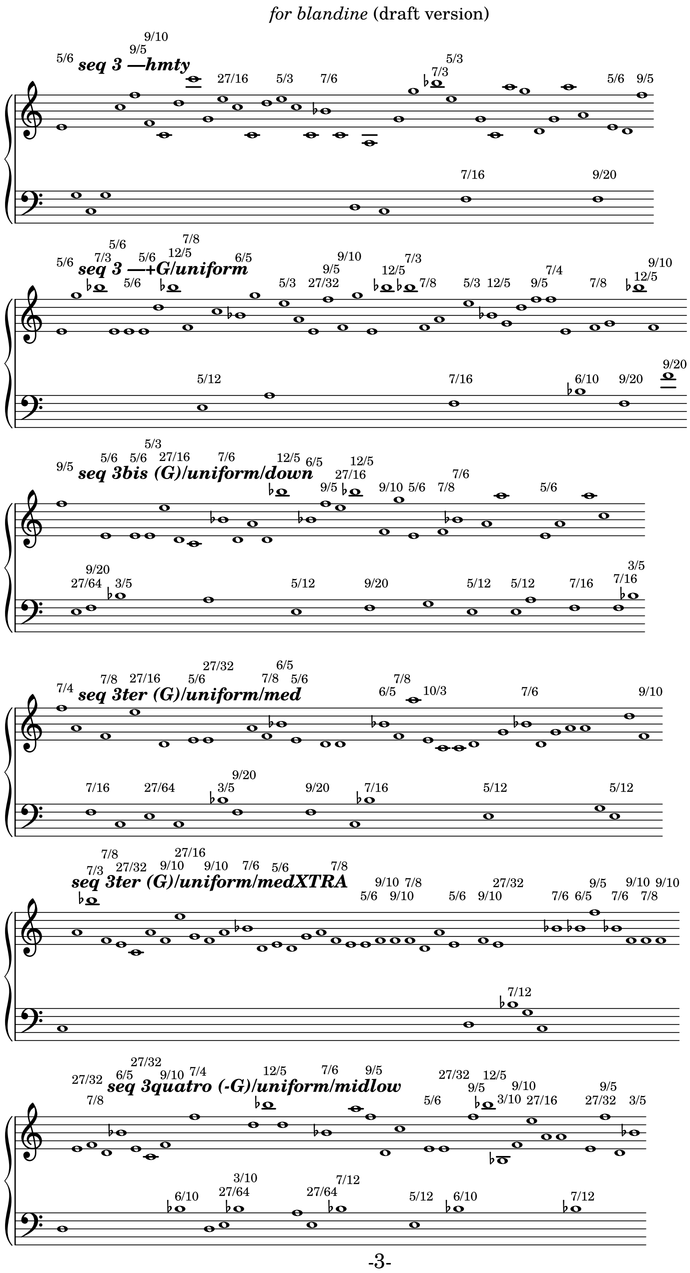

Fig. 7.1: *for blandine,* p.3, first initial version

7.2.2. Second version of the score

Manual re-write: phrasing pitch sequences by grouping notes into non-repeating phrases

To rework the score, I first coloured the different ratios to indicate explicitly on which keyboard each note was to be played: yellow corresponded to the upper manual and green to the lower. For practical/technical reasons, I left the rest of the notes uncoloured, so that the performer may choose on which keyboard to play a given note. I then grouped notes into non-repeating phrases of pitches — taking into consideration that differences in tuning would result in different pitch material.

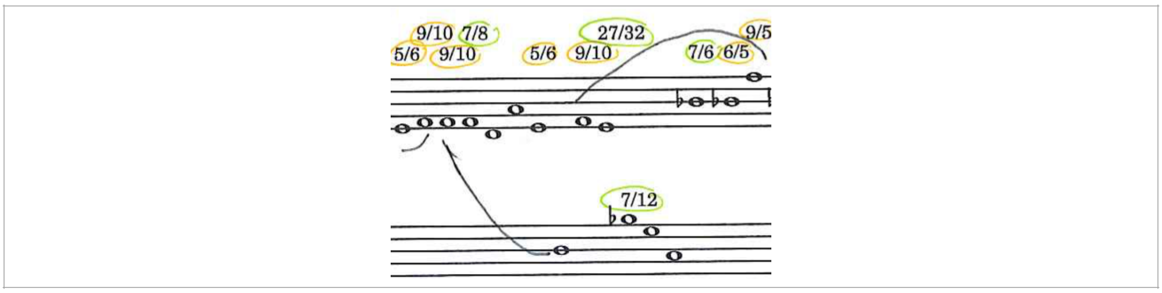

Fig. 7.2.a: *for blandine,* p.3, (zoomed-in) second version

This grouping was inspired by L. Couperin's *unmeasured preludes* notation (see Fig. 7.4), consisting of slurs and curves. It became my main compositional strategy and discovery in the piece, developing a rigid structure (determined notes in the score structured through slurs), while offering bounded possibilities of "free phrasing" to a performer. Rather than precise articulation instructions, I initially used this notation to suggest two main processes of differentiation of structural/non-structural notes: one for the performer, one for the listeners. Big slurs would define longer groups of notes and higher levels of perceptual units while shorter groups of notes smaller slurs define lower levels of perceptual units for the listeners. As a result of my algorithm and its random selection process, longer melodic sequences are found in the middle of the piece and shorter melodic sequences are present at the beginning and end of the piece.

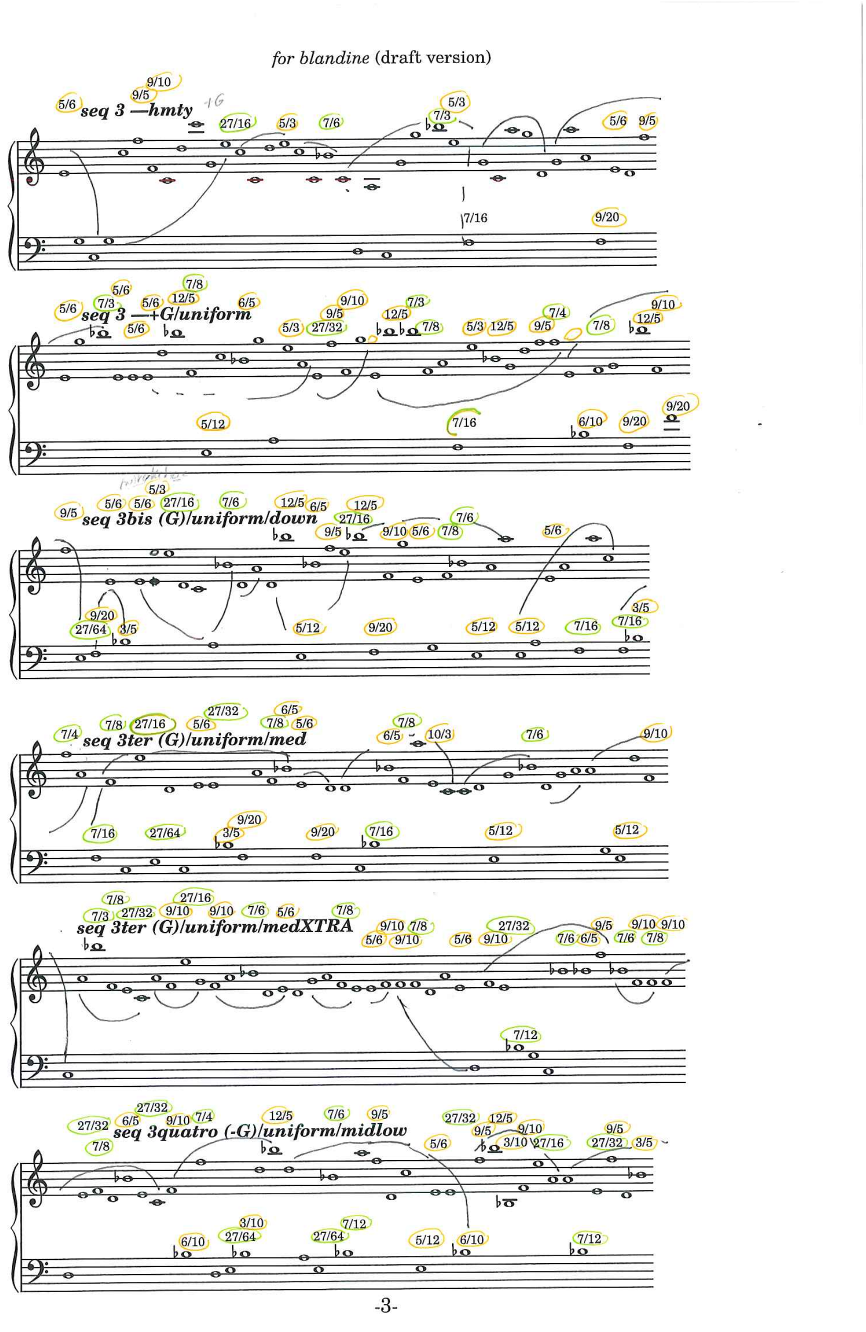

Fig. 7.2.b: *for blandine*, p.3, second version of with slurs and colors

7.2.3. Third version of the score

Maciej Skrzeczkowski's re-reading/annotating of the score

Maciej Skrzeczkowski, like Edgars Rubenis, worked in the unknown with the piece and we both found this process interesting. First, Maciej had never worked with just intonation before. In addition, I initially decided not to share information about the inner workings of the piece before inviting him to collaborate. For instance, the slurs ambiguously indicated harmonically significant notes in the score, suggesting to the performer that they need to *actively* undergo a process of differentiating notes into structural and non-structural categories. Specifically, the performer needs to go through a process of harmonic clarification of the work's pitch material and to interpret these slurs accordingly. I liken this idea to the analytical interpretation and transcription processes of Couperin's *unmeasured preludes*. Considering Maciej's understanding of early music and harmony,[112] I was expecting him to re-shape the overall harmonic/melodic movement in the piece — which he did with every practice. He could read the piece's slurs, indicating sustained notes (how long to hold one key) or how to group notes together into musical, melodic units, or separate successive groups of notes. The performer could make a combination of the latter options, consequently deciding on the overall pacing of the piece, and add more ornaments.

In his version of the score, his phrasings denotes how he tried to find harmonically significant notes, technically doable fingerings but also a wish not to "break the sound of the instrument" — which is what he had been mainly taught to do as a harpsichordist. Yet, when playing simultaneously the two keyboards all at once, 'not breaking the sound of the instrument' can be very difficult. As Maciej shared having felt "puzzled" by the piece, it is true that the slurs are at times anything but harmonically and technically logical for the performer. Thus, I can sense in his annotations to the score an internal dilemma between a wish to convey an overall, continuous phrasing and how these internal articulations would occasionally work against my original slurs, which tended to circumvent any sense of continuity and created recurring, intermittent disruptions.

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[112] Using slurs served as a bridge to communicate with early music performers and creating a bridge with my music.

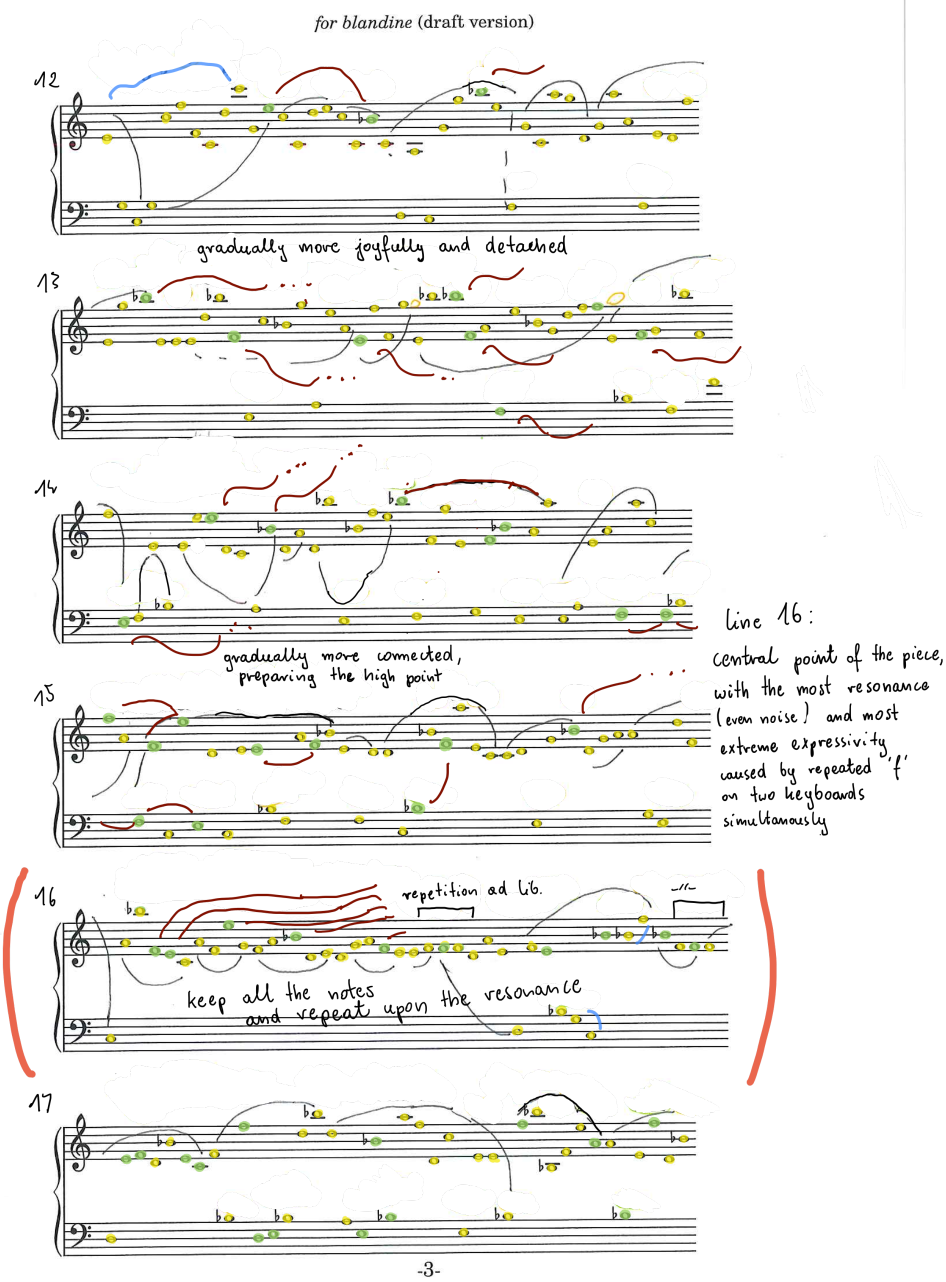

Fig. 7.3: *for blandine,* p.3, third version with additional slurs and comments by Maciej Skrzeczkowski.

7.3. Listening experiences of *for blandine and maciej*

7.3.1 Successive recordings of the piece's parts

Similar to the presentation of the three successive scores, sharing three successive recordings of the piece-in-progress expresses varying points of tension between algorithmic processes, early music convention, notation, and interpretation that are all central to the piece.

The first excerpt[113] comes from the first recording made of the harpsichord part, where the instrument was tuned to Maciej's customized, semi-equal temperament with just thirds. As such, the recording feels like a "mild" expression of its musical concept. Maciej's musicianship added a richness to the score and multiple ways to "read" the material. This was especially noticeable in terms of his rhythmic organization of the material; secondly, by his expressivity which added baroque gestures (e.g. ornamentation) into his interpretation. That said, the temperament of his instrument smoothed out the essential element of the piece, i.e. the different tunings of the two manuals. Lastly, because Maciej approached the work as an early music performer he inevitably played the score with a conventional harmonic interpretation, for instance, by emphasising bass notes as roots he weighted these notes to imply some presence of traditional harmony.

As we rehearsed the music we began discussing different ways to approach and interpret the slurs in the score. This included enhancing the ones Maciej played more spontaneously and conversely avoiding the ones that sounded more "automatic" or conventional. We discussed how to arrive at different types of fluidity when performing the piece by allocating fixed durations for each note (executing the score) and introducing long silences at unexpected times — this contrasted with Maciej's fear of ‘breaking the sound of the instrument’. In the end, his interpretation of the piece incorporated a variety of phrasing: in middle of the piece, he used more expressive phrasing (based on his interpretation of the material), however, at the end of the work he played the music in a more removed and detached way.

In parallel to this rehearsal process, I was working with simulations of the electronic part.[114] The electronics were intended to act as the bare skeleton of the music. Though in contrast to a "synthesized version" of the harpsichord part, this approach to composing the electronics owed much to my experiences with analysing and coding an algorithm based on *Arbor Vitae* and

[113] See Appendix 4.2.

[114] See Appendix 4.3.

experiments with a methodology of composing that dealt solely with tuning possibilities (without knowing how the piece would sound with the instrument and relying on my imagination and some degree of trust).

The second recording of the harpsichord part[115] was made a few days before the first performance of the piece. It is was the first time that Maciej finally got to try the piece with the right tuning (with the two manuals tuned differently). For both Maciej and I, the effect of this experience felt like a striking rediscovery of the piece, as the music spoke in a completely different way. Up to a certain degree, this unleashed a fresh look on the instrument's vibrancy as well. Nothing was lost from what or how Maciej had worked previously; simply, his understanding of the piece and way of playing the instrument became less "personal." In a way, both Maciej and I felt that the piece did not belong to either of us anymore. We also decided that we needed to continue to work separately with our own individual imaginations: Maciej on the harpsichord, and myself on the electronics. Maciej never heard the electronics before and the day of the first performance of the piece and I had not tried the electronics with Maciej either. Fortunately, this proved to be a very good decision, as it led our full discovery of the piece during the premiere. The combination between the two sound layers was somewhat "spacious," allowing the two parts to coexist in a rather balanced way.

7.3.2. Finalized version of the piece

By using unmeasured notation I had hoped to communicate to the listener a process of differentiation of non-repeating sequences of notes of varying lengths. However in practice, the delineation of sequences of notes as perceptual units was admittedly difficult to recognise. Instead, I now see that by mixing unmeasured notation with non-repeating groups of notes, a more paradoxical effect is created one that simultaneously gives rise to audible baroque-like phrasing but is contrasted by a "stochastic effect". Similar to the music heard in *Herma*, this "stochastic effect" in my piece is due to its overall speed and durations, the random ordering of notes and repetitions, and their wide range in the middle of the piece. For instance, the beginning of *for blandine and maciej* features the harpsichord part as if the performer was tuning the instrument; this is due to the repetition of the same two pitches and the simplicity of their harmonic relationship. Further on, the piece superficially (or accidentally) conveys something akin to baroque music. However, the piece eventually leads to a more "disorienting" experience, as the music reveals that it has structurally nothing to do with baroque music. This is noticeable when the tuning of the second keyboard is

[115] See Appendix 4.4.

introduced, the piece sounds very unfamiliar, or almost like something is going wrong — this is all the more amplified by the almost undetected pianissimo presence of sine tones in the background of the piece. But gradually towards the middle of the piece, the two sound layers coherently fall more into place. When the sine tones are finally heard on their own, they evoke a reduction of the previous musical experience of the harpsichord. The sine tones perhaps even convey some of the shared structural and mathematical avenues, which the two parts of the piece originated from, while still emphasizing their distinctness from the harpsichord. For instance, as sustained tones, the electronics are foreign to a notion of melodic phrasing, whereas the harpsichord's part consist of short sounds whose musicality depend primarily on phrasing.

7.4. Delving into the *Unmeasured preludes* notation

To better understand my compositional ideas and how I sought to combine the harpsichord and the electronics requires me to explain how I unexpectedly became interested in the history of Couperin's unmeasured preludes and Dan Tidhar's computational analysis of them.[116] I believe this "digression" supported my own ongoing compositional process — and has opened up more possibilities for future pieces.

7.4.1. Unmeasured scores as a probabilistic scenario: a personal account

After briefly encountering Louis Couperin's scores, working with unmeasured notation seemed like a logical step, largely because this form of notation expresses a rhythmical looseness or a form of "indeterminacy in relation to performance". This was also something I had experimented with in *for edgars,* and I noticed that it invited the performer to decipher and analyze the rhythmic-metrical relations of the notes present in the piece. Thus, each performer can find different ways of interpreting and even *transcribing* unmeasured scores — a point that appeared connected to my experience when reconstructing ("transcribing") Tenney's algorithm to *Arbor Vitae*. Additionally, I was attracted by the idea of considering the unmeasured scores in terms of visual and musical Gestalts. For me, slurs suggested differently sized melodic-harmonic *and* durational units, and, more hypothetically, a hierarchy of musical (rhythmical/harmonic) information.

Last but not least, I could see how this notation could reinforce the probabilistic dimension of my piece. To me, an *unmeasured score* may be seen as a probabilistic scenario that could potentially be analyzed through the lens of probabilities and/or as stochastic processes. I was

[116] Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes*. Technishe Universitaät Berlin, Berlin, December 2004.

curious where my piece would stand in this regard, as I had considered myself to be composing with both algorithm probabilistic processes and a form of probabilistic notation. I was also initially tempted to draw a parallels between unmeasuredness and stochastic processes, similar to Popoff's analysis of Cage's *Number Pieces*. However, this idea was rather naive if applied to Couperin's preludes. After all, the work differs from contemporary open scores and pieces of music, and as reminded by the harpsichordist and music theorist Philip Chang, "an infinite number of realizations [of Couperin's preludes] may exist, but not all of them will be tasteful and appropriate."[117] In fact, the rhythmic freedom of the performer is very bound up and contextualized in the inexpressible French *bon goût* or baroque musical conventions, requiring a meticulous harmonic analysis of the piece by the performer.

7.4.2. Musicological grounds of unmeasured preludes

7.4.2.1. Definition of a prelude

A prelude is "une composition libre, où l'imagination se livre à tout ce qui se présente à elle".[118] In his analysis of Couperin's unmeasured Prelude 7, Dan Tidhar interestingly highlights the similarities between Northern Indian *Ālap* music which helps establishing the "tonality" (the Rāg) of a performance and baroque preludes.[119] Indeed, a prelude introduces the key tonality of proceeding movements, pragmatically this enables a performer to test the tuning and condition of the instrument (lute, viola, or harpsichord),[120] allowing the performer to warm up while also preparing the listener to listen. And again similarly to *Ālap*'s improvisatory setting, Tidhar also reminds that preludes are generally performed with a certain amount of rhythmic-metrical looseness/flexibility, granting the preluding performer an important role and freedom of interpretation.[121]

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[117] Philip Chih-Cheng Chang. *Analytical and Performative Issues in Selected Unmeasured Preludes by Louis Couperin,* University of Rochester, New York, 2011, p.30.

[118] This is François Couperin (Louis's nephew)'s definition of a prelude: "a free composition where the imagination abandons itself to all that comes to it" François Couperin, *L'Art de toucher le clavecin,* A Paris, 1717, p. 61.

[119] Parallel made by Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes*. ibid. p.60.

[120] ibid. p.118.

[121] ibid., p.60.

7.4.2.2.Specificity of unmeasured preludes

The *unmeasured preludes* express this freedom of the performer through a particular system of notation, mainly developed by Louis Couperin. This notation delineates sequences of pitches with semibreves on the staves or more occasionally vertical bar lines; and groups pitches in further harmonic sub-collections with slurs. Couperin's "artistry and mastery of unmeasuredness"[122] has to do with the way his music suggests ordered groups of notes, articulations of harmonic events, and (which groups of) notes may be stressed or held — only vaguely indicating the length/duration of each note. Thus the performer defining the rhythmical pacing of the piece, adding silences, suspensions or ornamentation.

Fig. 7.4: One page of Couperin's prélude non mesuré (André Bauyn's manuscript transcription)[123]

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[122] Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes,* ibid., pp.30-31.

[123] Couperin, Louis. *Manuscrit Prélude non-mesuré,* Manuscrit Rés.Vm7 674–675 (ancienne cote : Vm7 1852 & Vm7 1862), dit manuscrit Bauyn [Prosper Bauyn d'Angersvillers], Bibliothèque nationale de France, copiste inconnu, daté d'après 1676 : 122 pièces pour clavecin, 4 pour orgue, 5 pour orchestre. Available on Musicologie Jean-Marc Warszawski, "Biographie Louis Couperin."2004. https://www.musicologie.org/Biographies/c/couperin_louis.html .

7.4.2.2.1. Unmeasured prelude interpretations: "harmonic rhythm"[124]

The tempo, phrasing, and agogic accents of Couperin's unmeasured preludes primarily depend on an interdependence between the specific timbre of the instrument: its limited resonance and the short decay of each struck note, and the harmonic structure of each groups of notes found in the score. Indeed, it is unanimously acknowledged that harmony is at the core of any interpretation of the unmeasured preludes. Thus, in the words of Dan Tidhar, "any decisions on part of the performer regarding groupings of notes and distribution of emphases will necessarily be perceived as some sort of metre providing orientation and notions of structure and direction of the music."[125]

7.4.2.2.2. Functions of slurs

Couperin's unmeasured notation always remained mysterious, first, in its historical origins and the accuracy of ability to transmit through time, but also as a precise guide for interpretation. Yet three main functions of the slurs are commonly admitted among performers and music analysts. First, slurs may indicate sustained notes, which are also harmonically the most crucial. These type of slurs determine the *relative length* of each note-head. The second type of slurs simply indicate grouping of notes, in a melodic context. The third enhances a separation or breath between successive notes or successive groups or notes. Deciphering the precise functions of these slurs in Couperin's music always requires a thorough analysis, illustrated in several extensive studies of individual Preludes.[126]

7.4.3.Computational analysis of unmeasured preludes

Among them, Dan Tidhar's (2005) analysis of Couperin's unmeasured Prelude Seven[127] particularly caught my attention. Tidhar used the tools to from computer science, music cognition and music analysis to build a computational grammar from the Prelude 7 and emulated one appropriate performance of the piece according to this grammar. Contrary to Popoff's analysis to Cage's *Number Pieces* as stochastic processes, using probabilities to emulate numerous performances of the piece and derive the piece's statistical analysis, Tidhar's methodology is

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124 Expression taken from Dan Tidhar, *Hierarchical and Deterministic Approach to Music Grammars and its Application toUnmeasured Preludes,* ibid. p.61.

125 Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes,* ibid., pp.61-62.

126 See Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes,* ibid**.** See also Chang, Philip Chih-Cheng. *Analytical and Performative Issues in Selected Unmeasured Preludes by Louis Couperin,* University of Rochester, New York, 2011.

127 Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application to Unmeasured Preludes,* ibid.

deterministic and cognitively motivated. His dissertation tries to illustrate that cognitively structural notes are those that are the most appropriate in terms of interpretation of the music. For him, "the [unmeasured prelude] score serves as a model *of* the composers intentions and as a model *for* its interpretation. The resulting music serves as a model *of* the interpretation (now a certain synthesis between the composer's and the interpreter's intentions); and this also serves as a model *for* the aesthetic process which is involved in the listening experience."[128] I find this way of considering notation and interpretation especially relevant in *for blandine and maciej,* as much as the way Tidhar highlights the role of the performer. In my opinion, an 'unmeasured preluder' has a very similar role to contemporary performers of open scores, such as Maciej in my piece.

Tidhar's approach to listening experiences is also one which intrigues me. To support his view upon the unmeasured preludes as a model for the "aesthetic process [involved] in the listening experience," Tidhar develops the interesting notion of *note impression*. It seems that the latter very much resonates with some compositional ideas I had for my piece. A *note impression* "describes the psycho-acoustical event in the listener's mind upon listening to an acoustic relation of a note. The cognitive content of a *note impression* [depends on] the listener's musical skills, their acquaintance with the piece, their momentary concentration level, etc." This notation considered as "a *representation* of the impressions of all the notes that [a piece] includes."[129] So a note impression is a mental image of a previously heard note or sound. I believe I have played a lot with *note impressions* in my own piece, thanks to the partially sequential combination of the harpsichord and electronic parts.

Based on this notion, Tidhar first finds in the Prelude the most "structurally significant notes," those which have the highest propensity to be retained by listeners. To do so, Tidhar goes through a detailed harmonic and structural analysis of the Prelude's notes, putting an emphasis on the bass notes, their modalities, how they suggest different harmonic units. Tidhar then compared the complexity and similarities of these harmonic events. All these rules found by the researcher to analyze the music allow him to deduce a "grammar," i.e. a Parse Tree, supposed to "[capture] the essential interpretational act of assigning different durations to the identically written notes"[130]. Indeed, at the last stage of his dissertation, Tidhar combines this Parse Tree with an interpretation model for deriving durations and this results in a *Weighted Tree*. In brief, each note is assigned a fixed duration according to how much it is considered musically significant. Typically, the more

[128] Dan Tidhar, *A Hierarchical and Deterministic Approach to Music Grammars and its Application toUnmeasured Preludes,* ibid. p.31.

[129] Dan Tidhar, p.25.

[130] Dan Tidhar, p.25.

significant a note in the Prelude is — according to Tidhar's grammatical rules — the longer they are sustained in his emulation of the piece and reciprocally. This enables Tidhar to emulate one single performance of the piece thanks to computational procedures and to assess/evaluate its musical relevance with other early music experts. For me, this evaluation is rather questionable as it consists in comparing the interpretation model with other rather vague types of time allocation for each notes: equal durations, random durations according to a uniform probability distribution. That said, for the time being, I will leave this aspect of Tidhar's analysis aside.

7.5. Comparing three methodologies and relations to unmeasuredness

To wrap things up, I will go through the different methodologies and relations developed around the unmeasured notation encountered in this section, starting with Couperin's.

Couperin's compositional process began from his free improvisation, a stream of an ordered set of notes transcribed into an unmeasured score, leading to a multiplicity of "free" interpretations with no fixed durations but constrained by musical conventions. Computations are obviously absent from Couperin's music but the *model* that this notation proposes unleashes a multiplicity of possible mathematical/algorithmic interpretations.

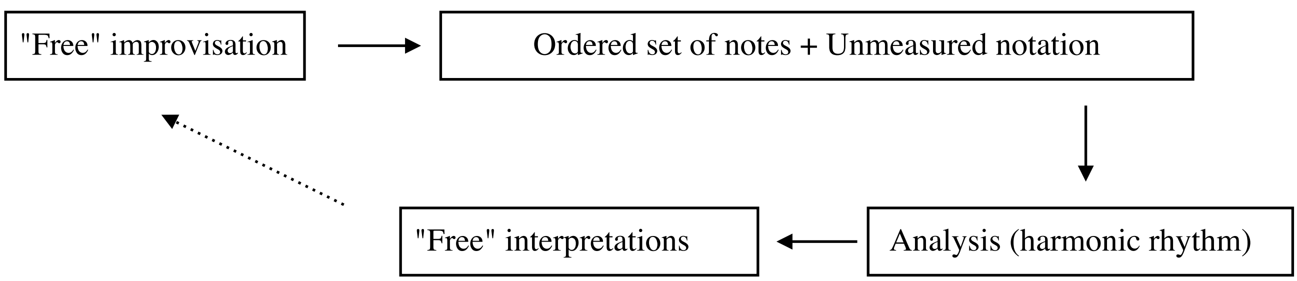

Fig 7.5: A flow-chart representing Couperin's unmeasured prelude, from composition to interpretation

One such 'algorithmic interpretation' of the Prelude is found in Tidhar's analysis. The latter starts from an unmeasured score, separating the notes into harmonic units, allowing a thorough harmonic and structural analysis of the piece and its "note impressions." This analysis is also the gateway for the derivation of mathematical rules and expressions from the Prelude, compiled into a deterministic computer program able to generate one determined interpretation of the piece.

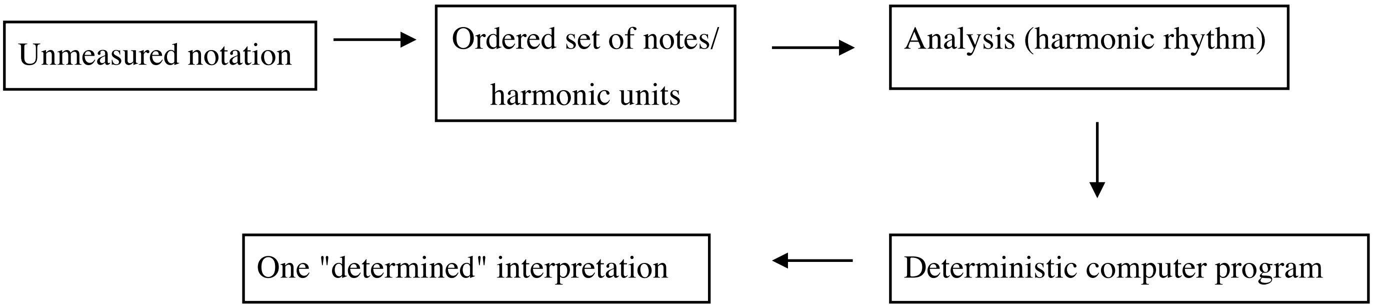

Fig 7.6: A flow-chart representing Tidhar's analysis of the Couperin's unmeasured prelude 7

Finally, my composition of *for blandine and maciej,* originally started from a computed, probabilistic generation of notes and an ordered set of notes. These notes were transformed into "notes impressions" by adding slurs and an unmeasured notation. The latter then led to a multiplicity of "free" interpretations, with no fixed, rigid pre-existing historical / aesthetic constraints but still required doing a harmonic analysis of the piece to find out its simple inner workings. The piece also includes a set of simultaneous, "unordered" sustained sine tones, acting like reminiscences of the harpsichord's "note impressions." Certainly, the interactions between the instrumental and electronic parts is still to be explored and enhanced — particularly the moments when the "notes impressions" on the harpsichord and "tones impressions" in the electronics ambiguously merge.

Working further on this aspect of the piece would deepen a compositional wish of mine: to create an intermediary space between the bare perception of harmonic relationships on the one hand, and, on the other hand, to have mentally inferred knowledge about the music's structure based on previous listening experiences.

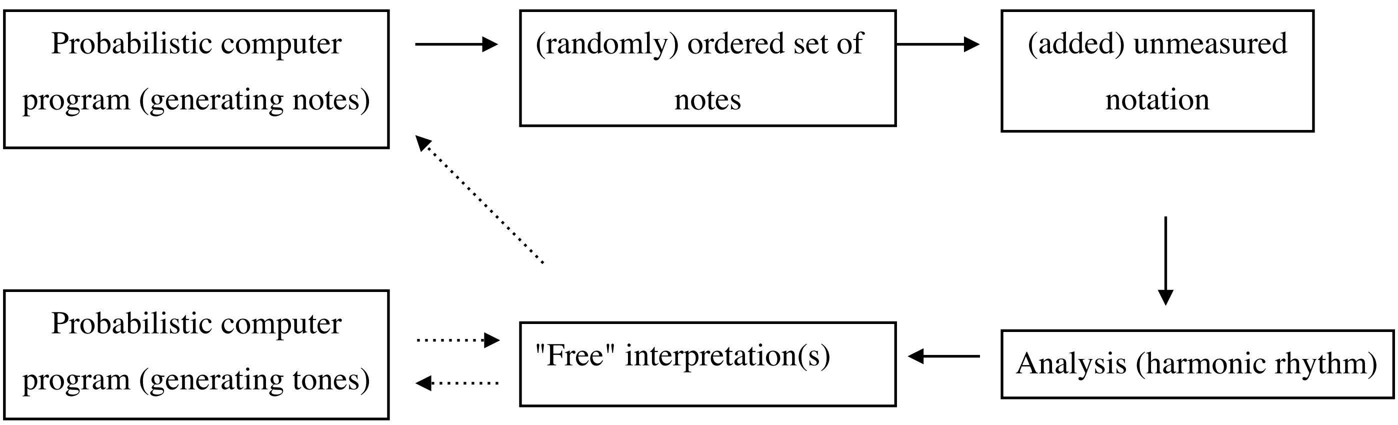

Fig 7.7: A flow-chart representing *for blandine and maciej,* from composition to performance

7.6. Conclusion

The piece *for blandine and maciej* illustrates the discovery of a compositional strategy, one existing between probabilities and unmeasuredness, and it also exemplifies a work that could be expanded and deepened. For instance, my use of slurs could be nested together in a much more harmonically complex and intentional way — reminiscent of Tenney's use of nestedness in *Arbor Vitae*'s.[131] These slurs could simultaneously allow different harmonic units to manifest themselves. Moreover, I would like this compositional strategy to be experienced with several performers, collectively deciphering the function of these slurs and complementing them with additional ones of their own. This leads me to also articulate the most important finding in this work: a way of relating with a performer. Beyond the music as such, I recognize that the musical collaboration is very crucial to me; this is also an interest I suggest with the titles of my works. Not being a musician myself creates an acute form of interdependency between the performers and I, one where we are on equal terms, learning and discovering the piece together. I find this way of working very motivating and humbling.

Last, but not least, through its successive steps, versions, recordings, the piece was an occasion for me (and Maciej) to retrace some memories of Blandine Verlet's "justesse" ("attunement" or "fine-tuning") and "touché," as well as the way she continuously refreshed and refined her approach to playing the instrument and music throughout the years.

> *… Tout au long de sa vie, on apprend, petit à petit, à toucher au plus juste la façon de traduire ses émotions par un instrument et puis on a l'impression que petit à petit on s'affine; alors on a peut-être tord, mais on a l'impression qu'on s'affine, donc c'est très agréable de retraduire des musiques que l'on a touchées pendant beaucoup d'années et d'avoir la sensation qu'on est plus apte à les traduire*.[132]

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[131] In that sense, without being able to articulate it clearly, I am intuiting a connexion between Couperin's slurs and Tenney's swells.

[132] Marc Zisman, *Blandine Verlet, une rencontre-podcast, Couperin "l'ami de toute une vie,"* 21 Mai 2012, Interview Audio, https://www.qobuz.com/fr-fr/info/magazine-actualites/rencontres/blandine-verlet-une-rencontre89772 An approximate translation "… Throughout one's life, we learn, little by little, how to "rightly touch" and translate our emotions through an instrument, and then we get the feeling that little by little, we refine ourselves or "fine tune"; so perhaps we are wrong, but we have the feeling that we refine ourselves, thus it is very pleasant to revisit some pieces of music that we "touched" for many years, and feeling that we are more able to translate them."

8. Conclusion (and opening)

In September 2020, I remember Richard Barrett asking me what were my artistic intentions. This question left me dumbfounded and intrigued, as I was not able to pinpoint the specificities of my work. From a very personal perspective, I could only intuit a question permeating to this project and my music, and this was one about "harmony" and what it may be, where it may begin, and where it may be found. Some answers were found gradually throughout the last two years, others were only found in the last few days, and the answer to others questions will hopefully follow in the future.

Importantly though, this project allowed me to take notice that my relation to harmony, mathematics and listening is quite peculiar. As a non-musician composer, computing rational intonation systems was my main entry to music theory and the only useful way I have found to deepen my understanding of musical intervals and harmony. In addition, I have partially developed my musicianship thanks to, or through, mathematics. Though still in its embryonic stages, my relation to mathematics is profound. Finding more clarity in mathematics allows me to find more clarity in my music and reciprocally. Certainly, finding this clarity will be a very long (and never-ending) process and this thesis was only a first step. But it has been a fruitful one: as I have begun to research and dissect some of the most simple, basic mathematical concepts involving probabilities and this has already spawned many compositional ideas.

Similarly, this research confirms my strong interest in efficient computational methods to create reduced, logical structures that apply to harmony and sound. Contrary to Flynt's assertion, I believe that attempting to combine considerations for abstract, mathematical structures *and* the sensory presence of sounds, is a compositional path worth pursuing. However, for me, this path requires one to follow it with flexibility; to circumvent being caught exclusively in either the perceptual pole, or (and perhaps especially) in the conceptual pole. Whether attached to music or mathematics, to quote the mathematician Albert Lautman, "our conceptions are never more than a provisional arrangement that allows the mind to go further forward."[133] In fact, this thesis is an example of such a provisional arrangement, one which has been woven from and owes much to the provisional arrangements made by others; thus provisional does not mean 'unimportant'.

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[133] Albert Lautman, *Mathematics, Ideas and the Physical Real,* Continuum International Publishing Group, 2011, p.88.

This leads me to the most significant realization of this project. Beneath my research question, that being the transition between mathematical concept, musical concept and percept, I had surprisingly missed an essential *link in the chain*. This missing link revolved around the relationships developed with people whose musicianship, music, words or teachings inspire my compositions. I have found in them precious friends and companions — ones who, sometimes unexpectedly or unknowingly, offered an immense and ineffable guidance.

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Appendix

1_arborVitae

- 1.1. Quatuor Bozzini, *Arbor Vitæ,* CQB 0806_NUM, January 1, 2008.
- 1.2. The reconstruction of the piece's algorithm on SuperCollider.
- 1.3. A synthesized realization of the piece made on 17.02.2022.

2_forEdgars

- 2.1. The recording of the piece performed by Edgars Rubenis on 09.04.2021.
- 2.2. Tuning of the strings on the guitar.
- 2.3. Tuning scheme with sine tones.
- 2.4. The score of the piece.
- 2.5. A video of a rehearsal of the piece at Studio Loos, April 2021.

3_diverseExamples

- 3.1. Iannis Xenakis's *Herma,* by Claude Helffer in *Iannis Xenakis: Chamber Music 1955 - 1990,* Believe Music, 2016.
- 3.2. John Cage's *Five* by Paul Hillier, Terry Riley, Theatre of Voices, Alan Bennett, Andrea Fullington, Paul Eliott, in *John Cage: Litany for the Whale,* Harmonia Mundi, 2002.
- 3.3. Clarence Barlow's *Approximating Pi* by Clarence Barlow, Self-release, 2010.
- 3.4. Catherine Christer Hennix's *The Electric Harpsichord* performed by Catherine Christer Hennix, Die Schachtel, 2010.
- 3.5. Michael Winter's *Approximating Omega* by Muirgene Leonore Gourgues, "Approximating Omega," Edition Wandelweiser Records, 2019.

4_forBlandineMaciej

- 4.1. The score of the piece.
- 4.2. An excerpt of the harpsichord part (not tuned) by Maciej Skrzeczkowski.
- 4.3. An excerpt of the harpsichord part (tuned)by Maciej Skrzeczkowski.
- 4.4. The electronics soloed.
- 4.5. The recording of the full piece performed by Maciej Skrzeczkowski at the Discussion Concert on 30.03.2022.